98 Calculus for Engineering Mathematics

1. $y = f(x),\quad y' = \dfrac{dy}{dx} = \lim_{\Delta x \to 0} \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ (or $y' = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$)

2. $y(x) = u(x)v(x) \quad \Rightarrow \quad y'(x) = u'(x)v(x) + u(x)v'(x)$

3. (a) $\int \dfrac{dy}{y} = \ln y + c$; $\dfrac{d}{dx}(\ln f(x)) = \dfrac{f'(x)}{f(x)}$

   (b) $\int \dfrac{dy}{(y-a)} = \ln(y-a) + c$; $\dfrac{d}{dx}(\ln(x-b)) = \dfrac{1}{x-b}$

   (c) $\int y^n dy = \dfrac{1}{(n+1)} y^{n+1} + c$; $\dfrac{d}{dx}(x^{n+1}) = (n+1)x^n$

   (d) $\int \dfrac{dy}{(y^2+1)} = \tan^{-1} y + c$; $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{(x^2+1)}$

   (e) $\int e^{ay} dy = \dfrac{1}{a} e^{ay} + c$; $\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

   (f) $\int \cos ay dy = \dfrac{1}{a} \sin ay + c$; $\dfrac{d}{dx}(\sin ax) = a\cos ax$

   (g) $\int \sin ay dy = -\dfrac{1}{a} \cos ay + c$; $\dfrac{d}{dx}(\cos ax) = -a\sin ax$

4. partial derivative

   Example：

   $z = f(x,y) = xy^2 + e^{xy}$

   $\dfrac{\partial z}{\partial x} = \dfrac{\partial f}{\partial x} = y^2 + e^{xy} \cdot y$

   $\dfrac{\partial z}{\partial y} = \dfrac{\partial f}{\partial y} = 2xy + e^{xy} \cdot x$

5. **Integration by parts (Examples)**

   $d(uv) = udv + vdu.$

   $uv = \int udv + \int vdu$

   $\int udv = uv - \int vdu$

   Example: $\int e^x \cos ax dx = ...$

5. Chain rule

   $z = f(x,y)$

   $dz = (\dfrac{\partial z}{\partial x})dx + (\dfrac{\partial z}{\partial y})dy \qquad \dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x} \cdot \dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y} \cdot \dfrac{\partial y}{\partial t}$

6. $\int \tan x dx = -\ln|\cos x| + c$

   $\int \cot x dx = \ln|\sin x| + c$

   $\int \sec x dx = \ln|\sec x + \tan x| + c$

   $\int \csc x dx = \ln|\csc x - \cot x| + c$