$v(\underset{\sim}{x})$ 是純量函數

(1) 一維梯度 $v(x)$

$$\nabla v = \frac{\partial v}{\partial x}\underset{\sim}{i}$$

$(-1, 0) \longleftarrow \boxed{} \longrightarrow (1, 0)$

方向導微 $(1,0) \Rightarrow \dfrac{\partial v}{\partial x}$,$(-1,0) \Rightarrow \dfrac{-\partial v}{\partial x}$

(2) 二維梯度 $v(x, y)$

$$\nabla v = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}\right) = \frac{\partial v}{\partial x}\underset{\sim}{i} + \frac{\partial v}{\partial y}\underset{\sim}{j}$$
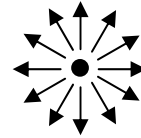
方向導微 $\dfrac{\partial v}{\partial n} = \nabla v \cdot \underset{\sim}{n}$ ($\underset{\sim}{n}$ 平面 360 度隨您選)

(3) 三維梯度 $v(x, y, z)$

$$\nabla v = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial z}\right) = \frac{\partial v}{\partial x}\underset{\sim}{i} + \frac{\partial v}{\partial y}\underset{\sim}{j} + \frac{\partial v}{\partial z}\underset{\sim}{k}$$

方向導微 $\dfrac{\partial v}{\partial n} = \nabla v \cdot \underset{\sim}{n}$ ($\underset{\sim}{n}$ 空間 4π 體角任您選)

※ $\dfrac{\partial v}{\partial n} = \lim\limits_{\varepsilon \to 0} \dfrac{v(\underset{\sim}{x}+\varepsilon\underset{\sim}{n}) - v(\underset{\sim}{x})}{\varepsilon}$ (v 函數在 n 方向導微)，其中 $\underset{\sim}{n}$ 為單位向量。

物理量(可想成溫度場)在 $\underset{\sim}{x}$ 處與 $\underset{\sim}{x}+\varepsilon\underset{\sim}{n}$ 處(鄰居，neighborhood)的差值除以兩點的距離($\varepsilon = \underset{\sim}{x}+\varepsilon\underset{\sim}{n} - \underset{\sim}{x}$)，即有該物理量在 $\underset{\sim}{n}$ 方向導微。

※多變數函數的 Taylor 展開式：

$$v\left(\underset{\sim}{x}+\varepsilon\underset{\sim}{n}\right) = v\left(\underset{\sim}{x}\right) + \frac{\partial v}{\partial x_1}\varepsilon n_1 + \frac{\partial v}{\partial x_2}\varepsilon n_2 + \frac{\partial v}{\partial x_3}\varepsilon n_3 + H.O.T.$$

Dot vs. Cross Product

https://www.youtube.com/watch?v=E34CftP455k

Gradient

https://www.youtube.com/watch?v=jUC-azOrLNs

Gradient 1(4:00)

https://www.youtube.com/watch?v=U7HQ_G_N6vo

Gradient of a scalar field

https://www.youtube.com/watch?v=OB8b8aDGLgE

*Grad - Grad, Div and Curl (1/3)

https://www.youtube.com/watch?v=ynzRyIL2atU

Vector Calculus - Divergence

https://www.youtube.com/watch?v=1Ve7gqf-zsc

Gradient of a Scalar Field - Dragonfly Education

https://www.youtube.com/watch?v=fZ231k3zsAA