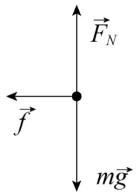
Chapter **6** ***Force and Motion – II***
賽車能在天花板上倒吊行駛嗎？

02. The free-body diagram for the player is shown next. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$.



We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,

$$\mu_k = f / F_N = (470 \text{ N})/(79 \text{ kg} \times 9.80 \text{ m/s2}) = 0.61 .$$

07. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$. (**a**) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \Rightarrow$$

$$F_N = (15)\sin 40° + (3.5)(9.8) = 44 \text{ (N)}.$$

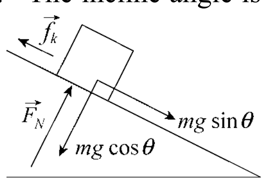
With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N. (**b**) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma$$

$$\Rightarrow a = \frac{(15)\cos 40° - 11}{3.5} = 0.14 \text{ (m/s}^2\text{)}.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

08. We first analyze the forces on the pig of mass m. The $+x$ direction is "downhill." The incline angle is θ. Application of Newton's second law to the x- and y- axes leads to



$$mg\sin\theta - f_k = ma$$

and $$F_N - mg\cos\theta = 0 .$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g\sin\theta - \mu_k\cos\theta = 0 .$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + (½)at^2 \Rightarrow t = \sqrt{2\ell / a} .$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell / a}}{\sqrt{2\ell / a'}} = \sqrt{\frac{a'}{a}} ,$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4(g\sin\theta - \mu_k\cos\theta).$$

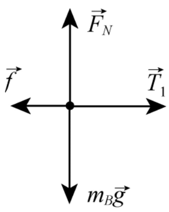
Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

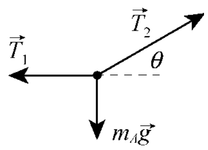
15. (**a**) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$N and $\mu_s F_N = (0.60)(12\text{N}) = 7.2$N. The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide. (**b**) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N\hat{\text{i}} + f\,\hat{\text{j}} = (-12 \text{ N})\,\hat{\text{i}} + (5.0 \text{ N})\,\hat{\text{j}},$$

where the axes are as shown on Fig. 6-25 of the text.

21. The free-body diagrams for block B and for the knot just above block A are shown next. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_{12}$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B ($W_B = 711$ N is the magnitude of $m_B\vec{g}$).





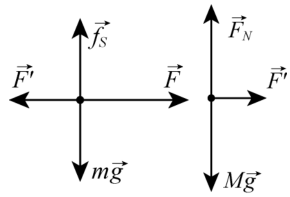
For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y axes for block B and then doing the same for the knot results in four equations:

$$T_1 - f_{s,\max} = 0, \quad F_N - W_B = 0,$$

$$T_2\cos\theta - T_1 = 0, \text{ and } \quad T_2\sin\theta - W_A = 0,$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 103 \text{ N} \approx 1.0\times10^2$ N.

29. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,max} = \mu_s F'$ where $\mu_s = 0.38$).



Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration.

$$F = m_{total}a \Rightarrow a = \frac{F}{m+M}.$$

This is equivalent to having analyzed the two blocks individually and then combined their eqs. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

$$F - F' = ma \Rightarrow F' = F - m\frac{F}{m+M}$$

and $f_s - mg = 0 \Rightarrow \mu_s F' - mg = 0$.

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s[1 - m/(m+M)]},$$

which we find to be $F = 4.9\times10^2$ N.

32. Using Eq. 6-16, we solve for the area $A = 2mg/(C\rho v_t^2)$, which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain A_{slow}/A_{fast} = (310 km/h)^2 / (160 km/h)^2 = 3.75 .

33. For the passenger jet $D_j = C\rho_1 A v_j^2/2$, and for the prop-driven transport $D_t = C\rho_2 A v_t^2/2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{(0.38)(1000^2)}{(0.67)(500^2)} = 2.3 .$$

37. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s F_N = \mu_s mg$. If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \Rightarrow R \ge \frac{v^2}{v_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{min} = \frac{v^2}{v_s g} = \frac{(8.1)}{(0.32)(9.8)} = 21 \text{ (m)}.$$

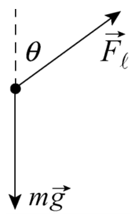
41. At the top of the hill, the situation is similar to that of S.P. 6-7 but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m\,(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem 6-7) and adapt Eq.6-19 accordingly. Thus we obtain

$$F_N = m\,(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37\times10^3 \text{ N}.$$

47. The free-body diagram (for the airplane of mass m) is shown below. We note that $\boldsymbol{F}_\ell$ is the force of aerodynamic lift and $\boldsymbol{a}$ points rightwards in the figure. We also note that $a = v^2/R$, where v = 480 km/h = 133 m/s. Applying Newton's law to the axes of the problem (+x rightward and +y upward) we obtain



$$\boldsymbol{F}_\ell \sin\theta = m\,v^2/R \text{ and } \boldsymbol{F}_\ell \cos\theta = mg,$$

where θ = 40°. Eliminating mass from these equations leads to

$$\tan\theta = v^2/gR,$$

which yields $R = v^2/g\tan\theta = 2.2\times10^3$ m.

49. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50)(9.80)(0.200)}{1.50}} = 1.81 \text{ (m/s)}.$$

52.• (**a**) We note that R (the horizontal distance from the bob to the axis of rotation) is the circumference of the circular path divided by 2π; therefore, $R = 0.94/2\pi = 0.15$ (m). The angle that the cord makes with the horizontal is now easily found:

$$\theta = \cos^{-1}(R/L) = \cos^{-1}(0.15/0.90) = 80^\circ.$$

The vertical component of the force of tension in the string is $T\sin\theta$ and must equal the downward pull of gravity (mg). Thus, $T = mg/\sin\theta = 0.40$ N. Note that we are using T for tension (not for the period). (**b**) The horizontal component of that tension must supply the centripetal force (Eq. 6-18), so we have $T\cos\theta = mv^2/R$. This gives speed v = 0.49 m/s. This divided into the circumference gives the time for one revolution: $\tau = 0.94/0.49 = 1.9$ (s).

57.• (**a**) Refer to the figure in the textbook accompanying S.P. 6-3 (Fig. 6-5). Replace f_s with f_k in Fig. 6-5(b). With θ = 60°, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma \text{ and } f = f_k = \mu_k F_N = \mu_k mg\cos\theta .$$

Thus, $a = g(\sin\theta - \mu_k\cos\theta) = 7.5 \text{ m/s}^2$.

(**b**) The direction of the acceleration $\boldsymbol{a}$ is down the slope. (**c**) Now the friction force is in the "downhill" direction (which is our positive direction) so that we obtain

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.5 \text{ m/s}^2.$$

(**d**) The direction is down the slope.

58.• (**a**) The x component of $\boldsymbol{F}$ tries to move the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

x direction: $F\cos\theta - f_s = 0$,

y direction: $F_N - F\sin\theta - mg = 0$.

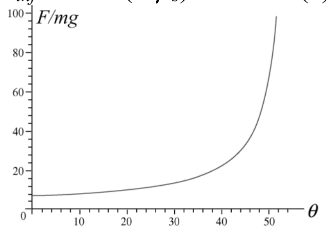
To be "on the verge of sliding" means $f_s = f_{s,max} = \mu_s F_N$. Solving these eqs. for F (actually, for the ratio of F to mg) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta}.$$

This is plotted below (θ in degrees). (**b**) The denominator of our expression (for F/mg) vanishes when

$$\cos\theta - \mu_s\sin\theta = 0 \Rightarrow \theta_{inf} = \tan^{-1}(1/\mu_s)$$

For $\mu_s = 0.70$, we obtain $\theta_{inf} = \tan^{-1}(1/\mu_s) = 55°$. (**c**) Reducing the coefficient means increasing the angle by the condition in part (b). (**d**) For $\mu_s = 0.60$, we have $\theta_{inf} = \tan^{-1}(1/\mu_s) = 59°$.



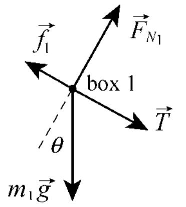
59.• (**a**) The x component of $\boldsymbol{F}$ contributes to the motion of the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Along the y direction, we have $F_N - F\cos\theta - mg = 0$ and along the x direction we have $F\sin\theta - f_k = 0$ (since it is not accelerating, according to the problem). Also, Eq. 6-2 gives $f_k = \mu_k F_N$. Solving these equations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta}.$$

(**b**) When $\theta < \theta_0 = \tan^{-1}\mu_s$, $\boldsymbol{F}$ will not be able to move the mop head.

61.• (**a**) Using $F = \mu_s mg$, the coefficient of static friction for the surface between the two blocks is $\mu_s = (12\text{ N})/(39.2\text{ N}) = 0.31$, where $m_t g = (4.0)(9.8) = 39.2$ N is the weight of the top block. Let $M = m_t + m_b = 9.0$ kg be the total *system* mass, then the maximum horizontal force has a magnitude $Ma = M\mu_s g = 27$ N. (**b**) The acceleration (in the maximal case) is $a = \mu_s g = 3.0\text{ m/s}^2$.

68.• The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\boldsymbol{F}_{N2}$ is the normal force on box 2 (the uncle box), $\boldsymbol{F}_{N1}$ is the normal force on the aunt box (box 1), $\boldsymbol{f}_1$ is kinetic friction force on the aunt box, and $\boldsymbol{f}_2$ is kinetic friction force on the uncle box. Also, $m_1 = 1.65$ kg is the mass of the aunt box and $m_2 = 3.30$ kg is the mass of the uncle box (which is a lot of ants!). For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:



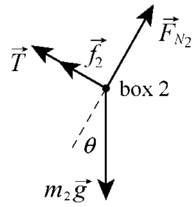
$$m_2 g\sin\theta - f_2 - T = m_2 a,$$

$$F_{N2} = m_2 g\cos\theta,$$

$$m_1 g\sin\theta - f_1 + T = m_2 a,$$

and $$F_{N1} = m_1 g\cos\theta,$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 F_{N1}$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 F_{N2}$ where $\mu_2 = 0.113$), fully describe the dynamics of the system. (**a**) We solve the above equations for the tension and obtain



$$T = \frac{m_1 m_2 g}{m_1 + m_2}(\mu_1 - \mu_2)\cos\theta = 1.05\text{ N}.$$

(**b**) These equations lead to an acceleration equal to

$$a = g\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} g\cos\theta = 3.62\text{ m/s}^2.$$

(**c**) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

90.• For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\boldsymbol{P}$ (which is upward with magnitude $P\sin\theta$). Since there is no acceleration in the vertical direction, we must have

$$F_N = P\sin\theta - mg,$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k (P\sin\theta - mg).$$

Choosing $+x$ rightward, Newton's second law leads to

$$P\cos\theta - f_k = ma \Rightarrow$$

$$a = \frac{P\cos\theta - \mu_k(P\sin\theta - mg)}{m},$$

which yields $a = 3.4\text{ m/s}^2$ when $\mu_k = 0.40$ and $m = 5.0$ kg.

97.• The coordinate system we wish to use is shown in Fig. 5-18 in the textbook, so we resolve this horizontal force into appropriate components. (**a**) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain

$$F\cos\theta - f_k - mg\sin\theta = ma,$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

Using $f_k = \mu_k F_N$, these equations lead to

$$a = (F/m)(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta),$$

which yields $a = -2.1\,\text{m/s}^2$, or $|a| = 2.1\,\text{m/s}^2$, for $\mu_k = 0.30$, $F = 50\,\text{N}$ and $m = 5.0\,\text{kg}$. (**b**) The direction of $\boldsymbol{a}$ is down the plane. (**c**) With $v_0 = +4.0\,\text{m/s}$ and $v = 0$, Eq. 2-16 gives

$$\Delta x = -4.0^2/[(2)(-2.1)] = 3.9\ (\text{m}).$$

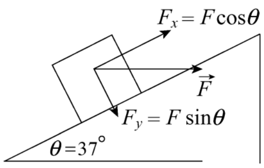
(**d**) We expect μ_s, not μ_k; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,max} = \mu_s F_N = \mu_s (F\sin\theta + mg\cos\theta),$$

which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10\ \text{N}.$$

(the fact that this is positive reinforces our suspicion that $\boldsymbol{f}_s$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,max}$ then it stays at that location.



***Ex.3-2*, *Pb. 6-25*.**

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

•急速時代的變格：急速科學, Discovery Channel

friction,摩擦/摩擦力; frictional force,摩擦力; negative lift,負升力; static/kinetic frictional force,靜/動摩擦力; coefficient of static friction,靜摩擦係數; coefficient of kinetic friction,動摩擦係數; drag force,拖曳力; drag coefficient,拖曳係數; effective cross-sectional area,等效截面積; terminal speed,終端速率; uniform circular motion,等速率圓周運動; centripetal acceleration/force,向心加速度/力; center of curvature,曲率中心; banked,有坡面的; bobsled,大雪橇; skydiving,特技跳傘; spread eagle,大鵬展翅(鯤魚化為大鵬鳥，一飛數萬里。用以比喻前程遠大，不可限量。); Dare Devil,蠻勇之人; Grand Prix 國際汽車大獎賽; pit,(賽車中途的)加油站, 修理站;

•**備忘錄**

重點整理－第 6 章 力與運動—II

摩擦力 $\boldsymbol{f}$：當作用力$\boldsymbol{F}$試著沿著表面滑動物體時，從表面來的**摩擦力**作用於此物體上，摩擦力平行表面並且其方向為阻止滑動，摩擦力導源於物體與接觸表面間的鍵結。若物體尚未滑動時，摩擦力為**靜摩擦力$\boldsymbol{f}_s$**；若物體滑動時，則為**動摩擦力$\boldsymbol{f}_k$**。

摩擦力之三性質 ♦性質1.若物體尚未運動，則靜摩擦力與施力$\boldsymbol{F}$平行表面的分量兩者大小相等，而$\boldsymbol{f}_s$的方向為與該分量相反。♦性質2. $\boldsymbol{f}_s$有一最大值$\boldsymbol{f}_{s,max}$，其為 $f_{s,max} = \mu_s F_N$ ，式中μ_s稱為**靜摩擦係數**而F_N為正向力的大小。若$\boldsymbol{F}$平行表面的分量大於$\boldsymbol{f}_{s,max}$，則物體開始於表面上滑動。♦性質3.若物體開始於表面上滑動，則摩擦力大小立即減小至一定值f_k，其為 $f_k = \mu_k F_N$ ，其中μ_k為**動摩擦係數**。

拖曳力 D 當物體與空氣(或其它流體)作相對運動時，其受**拖曳力**作用，此力阻止相對運動而且指向流體相對於物體之流動方向，拖曳力的大小藉由實驗決定的**拖曳係數** C 與相對速率 v 產生關聯，其為 $D = (½)C\rho A v^2$，

式中ρ為流體密度(每單位體積之質量)，A 為物體的等效截面積(取垂直相對速度 $\boldsymbol{v}$ 之截面積)。

終端速率 v_t 當鈍形物體在空氣中下落夠深時，物體所受的拖曳力與其所受的重力 $\boldsymbol{F}_g$ 兩者大小變為相等，之後物體以固定的**終端速率** v_t 下落，其為 $v_t = \sqrt{2F_g / C\rho A}$ 。

等速率圓周運動 若質點以等速率 v 於半徑為 R 之圓周或圓弧上運動，則稱為質點作**等速率圓周運動**，此質點於是有向心加速度，其大小為 $a = v^2/R$；此加速度是由於作用於質點之淨向心力而產生，此向心力大小為 $f = m v^2/R$，

其中m為質點之質量；向量$\boldsymbol{a}$及$\boldsymbol{F}$均指向質點路徑的曲率中心。