Chapter 7 *Kinetic Energy and Work*

該模擬應如何程序化以便給實習醫師硬膜外療程正確的感覺？

01. With speed $v = 1.12\times10^4$ m/s, we find $K = (½)mv^2 = (½)(2.9\times10^5)(1.12\times10^4)^2 = 1.8\times10^{13}$ (J).

03. (**a**) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = (v_0^2 + 2a\Delta x) = 2.9\times10^7 \text{ m/s}.$$

(**b**) The initial kinetic energy is

$$K_i = (½)mv_0^2 = 4.8\times10^{-13} \text{ J}.$$

The final kinetic energy is

$$K_f = (½)mv^2 = 6.9\times10^{-13} \text{ J}.$$

The change in kinetic energy is

$$\Delta K = (6.9\times10^{-13} - 4.8\times10^{-13}) \text{ J} = 2.1\times10^{-13} \text{ J}.$$

08. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F}\cdot\vec{d} = (210\,\hat{\mathrm{i}} - 150\,\hat{\mathrm{j}})\cdot(15\,\hat{\mathrm{i}} - 12\,\hat{\mathrm{j}})$$
$$= (210)(15) + (-150)(-12) = 5.0\times10^3 \text{ (J)}.$$

11. We choose $+x$ as the direction of motion (so $\boldsymbol{a}$ and $\boldsymbol{F}$ are negative-valued). (**a**) Newton's second law readily yields $\boldsymbol{F} = (85 \text{ kg})(-2.0 \text{ m/s}^2\, \mathbf{i})$ so that

$$F = |\vec{F}| = 1.7\times10^2 \text{ N}.$$

(**b**) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -\frac{(37)^2}{2(-2.0)} = 3.4\times10^2 \text{ (m)}.$$

Alternatively, this can be worked using the work-energy theorem. (**c**) Since $\boldsymbol{F}$ is opposite to the direction of motion (so the angle ϕ between $\boldsymbol{F}$ and $\boldsymbol{d}$ (= Δx **i**) is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times10^4$ J. (**d**) In this case, Newton's second law yields $\boldsymbol{F} = (85 \text{ kg})(-4.0 \text{ m/s}^2\, \mathbf{i})$ so that

$$F = |\vec{F}| = 3.4\times10^2 \text{ N}.$$

(**e**) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37)^2}{2(-4.0)} = 1.7\times10^2 \text{ (m)}.$$

(**f**) The force $\boldsymbol{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

17. (**a**) We use $\boldsymbol{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $g/10$ upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11mg/10$. Since the force $\boldsymbol{F}$ and the displacement $\boldsymbol{d}$ are in the same direction, the work done by $\boldsymbol{F}$ is

$$W_F = Fd = (11/10)mgd$$
$$= (11/10)(72 \text{ kg})(9.80 \text{ m/s}^2)(15 \text{ m}) = 1.164\times10^4 \text{ J},$$

which (with respect to significant figures) should be quoted as 1.2×10^4 J. (**b**) The force of gravity has magnitude mg and is opposite in direction to the displacement. Thus, using Eq.7-7, the work done by gravity is

$$W_g = -mgd = -(72 \text{ kg})(9.80 \text{ m/s}^2)(15 \text{ m})$$
$$= -1.058\times10^4 \text{ J},$$

which should be quoted as -1.1×10^4 J. (**c**) The total work done is $W = 1.164\times10^4 \text{ J} - 1.058\times10^4 \text{ J} = 1.06\times10^3$ J. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to 1.1×10^3 J) is her final kinetic energy. (**d**) Since $K = (½)mv^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06\times10^3)}{72}} = 5.4 \text{ (m/s)}.$$

18. (**a**) Using notation common to many vector capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, −(3.00)(9.80)], [0.500 ∠30.0º]) = 1.31 J. (**b**) Eq.7-10 (along with Eq.7-1) then leads to

$$v = \sqrt{2(1.31 \text{ J})/(3.00 \text{ kg})} = 0.935 \text{ m/s}.$$

25. We make use of Eq. 7-25 and Eq. 7-28 since the block is stationary before and after the displacement. The work done by the applied force can be written as

$$W_a = -W_s = (½)k(x_f^2 - x_i^2).$$

The spring constant is $k = 80/0.020 = 4.0\times10^3$ (N/m). With $W_a = 4.0$ J, and $x_i = -2.0$ cm, we have

$$x_f = \pm\sqrt{\frac{2W_a}{k} + x_i^2} = \pm\sqrt{\frac{2(4.0)}{4.0\times10^3} + (-0.020)^2}$$
$$= \pm0.049 \text{ (m)} = \pm4.9 \text{ (cm)}.$$

29. (**a**) As the body moves along the x axis from $x_i = 3.0$ m to $x_f = 4.0$ m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} (-6x)\,dx = -3(x_f^2 - x_i^2)$$
$$= -3(4.0^2 - 3.0^2) = -21 \text{ (J)}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$\Delta K = (½)m(v_f^2 - v_i^2) = W,$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21)}{2.0} + (8.0)^2} = 6.6 \text{ (m/s)}.$$

(**b**) The velocity of the particle is $v_f = 5.0$ m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the theorem leads to

$$-3(x_f^2 - x_i^2) = (½)m(v_f^2 - v_i^2).$$

Thus, $$x_f = \sqrt{-\frac{m}{6}(v_f^2 - v_i^2) + x_i^2}$$

$$= \sqrt{-\frac{2.0}{6}(5.0^2 - 8.0^2) + 3.0^2} = 4.7 \text{ (m)}.$$

31. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = (20\ \text{m/s}^2)/(8.0\ \text{m}) = 2.5\ \text{s}^{-2}.$$

The force on the brick is in the $+x$ direction and, according to Newton's second law, its magnitude is given by $F = a/m = (\alpha/m)x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F dx = \frac{\alpha}{m}\int_0^{x_f} x dx = \frac{\alpha}{2m}x_f^2$$
$$= (2.5)(2)^{-1}(10)^{-1}(8.0)^2 = 8.0\times10^2\ \text{(J)}.$$

35. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = dx/dt = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is $v_i = 3.0$ m/s and the speed at $t = 4$ s is $v_f = 19$ m/s. The change in kinetic energy for the object of mass $m = 3.0$ kg is therefore

$$\Delta K = (½)m(v_f^2 - v_i^2) = 528\ \text{J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3\times10^2$ J.

41. The power associated with force $\boldsymbol{F}$ is given by $P = \boldsymbol{F}\cdot\boldsymbol{v}$, where $\boldsymbol{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F}\cdot\vec{v} = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37°$$
$$= 4.9\times10^2\ \text{W}.$$

46. (**a**) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero. (**b**) The rate is given by $P = \boldsymbol{F}\cdot\boldsymbol{v} = -Fv$, where the minus sign corresponds to the fact that $\boldsymbol{F}$ and $\boldsymbol{v}$ are anti-parallel to each other. The magnitude of the force is given by $F = kx = (500$ N/m)(0.10 m) = 50 N, while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = (½)mv^2 + (½)kx^2$$
$$= (½)(0.30\ \text{kg})v^2 + (½)(500\ \text{N/m})(0.10\ \text{m})^2$$

which gives $v = 7.1$ m/s. Thus

$$P = -Fv = -(50\ \text{N})(7.1\ \text{m/s}) = -3.5\times10^2\ \text{W}.$$

50.• (**a**) The compression of the spring is $d = 0.12$ m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\ \text{kg})(9.80\ \text{m/s}^2)(0/12\ \text{m}) = 0.29\ \text{J}.$$

(**b**) The work done by the spring is, by Eq. 7-26,

$$W_2 = -(½)kd^2 = -(½)(250\ \text{N/m})(0.12\ \text{m})^2 = -1.8\ \text{J}.$$

(**c**) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15).

$$\Delta K = 0 - (½)mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29-1.8)}{0.25}} = 3.5\ \text{(m/s)}.$$

(**d**) If we instead had $v_i' = 7$ m/s, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2}mv_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2}kd'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = (12.048)^{1/2} = 3.471$ (m/s) and $v_i' = 6.942$ m/s).

62.• Using Eq. 7-8, we find

$$W = \vec{F}\cdot\vec{d} = F(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}})\cdot(x\,\hat{\text{i}} + y\,\hat{\text{j}})$$
$$= Fx\cos\theta + Fy\sin\theta,$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

65.• One approach is to assume a "path" from $\boldsymbol{r}_i$ to $\boldsymbol{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_x dx = \int_2^{-4} 2x\, dx + \int_3^{-3} 3\, dx$$

with SI units understood. Thus, we obtain $W = 12 - 18 = -6$ (J).

70.• (**a**) To hold the crate at equilibrium in the final situation, $\boldsymbol{F}$ must have the same magnitude as the horizontal component of the rope's tension $T\sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}(4.00/12.0) = 19.5°.$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is

$$T = (230)(9.80)/\cos 19.5° = 2391\ \text{(N)}$$

and $F = (2391)\sin 19.5° = 797$ (N). An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution. (**b**) Since there is no change in kinetic energy, the net work on it is zero. (**c**) The work done by gravity is $W_g = \boldsymbol{F}_g\cdot\boldsymbol{d} = -mgh$, where $h = L(1-\cos\theta)$ is the vertical component of the displace- ment. With $L = 12.0$ m, we obtain $W_g = -1547$ J which should be rounded to three figures: -1.55 kJ. (**d**) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since $\cos 90° = 0$). (**e**) The implication of the previous three parts is that the work due to $\boldsymbol{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g = 1.55$ kJ. (**f**) Since $\boldsymbol{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

73.• A convenient approach is provided by Eq. 7-48. $P = Fv$ = (1800 kg+4500 kg)(9.8 m/s^2)(3.80 m/s) = 235 kW. Note that we have set the applied force equal to the weight in order to maintain constant velocity (zero acceleration).

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重點整理－第 7 章 動能與功

合力對物體(質點)所作的淨功

等於其動能改變量 $W_{ext} = K_f - K_i = \Delta K$

動能 與質量 m 及速率 v (v 遠小於光速)之質點運動有關的動能 K 為

$$K = (½)mv^2 \quad \textbf{(動能)}. \quad (7\text{-}1)$$

功 W 功是經由作用於物體之力移轉入物體的能量或從此物體移轉出的能量，能量移轉入物體為**正功**，能量從物體移轉出則為**負功**。

定力作功 定力 $\vec{F}$ 對質點於位移 $\vec{d}$ 內所作的功為

$$W = Fd\cos\phi = \vec{F}\cdot\vec{d} \quad \textbf{(功,定力)} \quad (7\text{-}7,8)$$

其中 ϕ 為 $\vec{F}$ 和 $\vec{d}$ 間的夾角。只有 $\vec{F}$ 沿著位移 $\vec{d}$ 方向的分量才能對物體作功。當兩個或以上的力作用於一物體時，**淨功**為各別力所作的功的和，亦等於這些力之淨力所作的功。

功與動能 質點之動能改變量 ΔK 與對質點所作的淨功 W 的關係為

$$\Delta K = K_f - K_i = W \quad \textbf{(功–動能定理)}, \quad (7\text{-}10)$$

其中 K_i 為質點之初動能，而 K_f 為作功後之動能，7-10 式重新整理後得

$$K_f = K_i + W. \quad (7\text{-}11)$$

重力作功 當物體位移 $\vec{d}$ 時，作用於質量 m 的似質點物體之重力 $\vec{F}_g$ 所作的功 W_g 為

$$W_g = mgd\cos\phi = -mg\Delta y, \quad (7\text{-}12)$$

其中 ϕ 為 $\vec{F}_g$ 和 $\vec{d}$ 間的夾角，Δy 為高度改變量。

物體升高或降低時所作的功 當似質點物體升高或降低時，外施力所作的功 W_a 與重力所作的功 W_g 及物體動能改變量 ΔK 有關聯，即

$$\Delta K = K_f - K_i = W_a + W_g. \quad (7\text{-}15)$$

假如初動能等於末動能時，7-15 式簡化為

$$W_a = -W_g, \quad (7\text{-}16)$$

這表示施力移轉入物體的能量與重力從物體移轉出之能量相等。

彈力 彈簧產生的力 $\vec{F}_{sp}$ 為

$$\vec{F}_{sp} = -k\vec{d} \quad \textbf{(虎克定律)}, \quad (7\text{-}20)$$

式中 $\vec{d}$ 是彈簧自由端離其於彈簧處於**鬆弛狀態**(未壓縮亦未伸長)時的位置之**位移**，而 k 為**彈簧常數**(彈簧硬度的測量)。若 x 軸沿著彈簧，且以彈簧在鬆弛狀態時，彈簧的自由端為原點，則 7-20 式可寫為

$$F_x = -kx, \quad \textbf{(虎克定律)}. \quad (7\text{-}21)$$

彈力為變力，隨著彈簧自由端的位移而改變。

彈力所作的功 假如物體與彈簧自由端連接，當物體從初位置 x_i 運動到末位置 x_f 時，彈力對此物體所作的功 W_{sp} 為

$$W_{sp} = (½)kx_i^2 - (½)kx_f^2. \quad (7\text{-}25)$$

若 $x_i = 0$ 及 $x_f = x$，則 7-25 式變為

$$W_{sp} = -(½)kx^2. \quad (7\text{-}26)$$

變力所作的功 當作用於似質點物體之力 $\vec{F}$ 與物體的位置有關時，物體從座標為(x_i, y_i, z_i)的初位置 $\mathbf{r}_i$ 運動到座標為(x_f, y_f, z_f)的末位置 $\mathbf{r}_f$，則該力對物體所作的功須藉積分求得。若設分量 F_x 只與 x 有關但與 y 或 z 無關、分量 F_y 只與 y 有關但與 x 或 z 無關，以及分量 F_z 只與 z 有關但與 x 或 y 無關，則所作功為

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y'_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz. \quad (7\text{-}36)$$

假如 $\vec{F}$ 只有 x 方向分量，則 7-36 式簡化為

$$W = \int_{x_i}^{x_f} F_x\,dx. \quad (7\text{-}32)$$

功率 由力產生的功率為力對物體作功的速率。假如力於時距 Δt 內作功 ΔW，則該力在該時距內產生的平均功率為

$$P_{av} = \Delta W / \Delta t, \quad (7\text{-}42)$$

瞬時功率為作功的時變率

$$P = dW / dt, \quad (7\text{-}43)$$

假如力與物體運動方向的夾 ϕ 角，則瞬時功率為

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}, \quad (7\text{-}47,48)$$

其中 $\vec{v}$ 為物體的(瞬時)速度。

time interval,時距/時間間隔;energy,能量; (principle of) energy conservation,能量守恆(原理); kinetic energy,動能; work,功; net work,淨功; joule (J),焦耳; work–kinetic energy theorem,功–動能定理; gravitational force,重力; spring force,彈力; varying force,變力; Hooke's law,虎克定律; force constant (of spring),(彈)力常數; spring constant,彈簧常數; applied force,施力; power,功率; instantaneous/average work,瞬時/平均功率; practicing doctor, 實習醫師; anesthetic fluid,麻醉液; epidural (硬膜外) space (腔) /procedure (療程), spinal canal,椎管; spinal cord,脊椎神經/脊髓; ligament,韌帶; spinous,刺狀的; block,積木; cart,運貨車; crate,條板箱; helicopter,直升飛機; ladle,杓子; legendary,傳奇的; locomotives,機車/火車頭; luge,(競賽用的)仰臥滑行小雪橇; stoop,屈身彎腰; transfer,移轉; •**備忘錄**•