Chapter **11** *Rolling, Torque, and Angular Momentum*

01. The initial speed of the car is v = (80.0)(1000)/3600 = 22.2 (m/s). The tire radius is R = 0.750/2 = 0.375 (m). (**a**) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = v_{cm,0}/R = 22.2/0.375 = 59.3 \text{ (rad/s)}.$$

(**b**) With θ = (30.0)(2π) = 188 rad and ω = 0, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \Rightarrow$$

$$|\alpha| = 59.3^2/(2\times 188) = 9.31 \text{ (rad/s}^2).$$

(**c**) Eq.11-1 gives R = 70.7 m for the distance traveled.

05. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is $K_{rot} = 4(½)I\omega^2$, where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = (½)Mv^2+4(½)I\omega^2$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{rot}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}.$$

For a uniform disk (relative to its center of mass) $I = (½)mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is $4I\omega^2 = 4(½)mR^2(v/R)^2 = 2mv^2$ and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50}$$
$$= 0.020 .$$

The wheel radius cancels from the eqs. and is not needed in the computation.

08. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to <*cf. Prob. 8-8 & 21*>

$$U_{release} = K_{top} + U_{top} \text{ or}$$
$$mgh = \tfrac{1}{2} mv_{cm}^2 + \tfrac{1}{2} I\omega^2 + mg(2R).$$

Substituting $I = 2mR^2/5$ (Table 10-2(f)) and $\omega = v_{cm}/r$ (Eq. 11-2), we obtain

$$mgh = \tfrac{1}{2} mv_{cm}^2 + \tfrac{1}{2}(\tfrac{2}{5})mr^2\omega^2 + 2mgR$$
$$\Rightarrow gh = \tfrac{7}{10} v_{cm}^2 + 2gR .$$

where we have canceled out mass m in that last step. (**a**) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction (+y downward) leads to

$$mg = ma_r \quad \Rightarrow \quad g = \frac{v_{cm}^2}{R - r},$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance R–r from the center of the loop). Plugging the result $v_{cm}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \tfrac{7}{10} g(R-r) + 2gR ,$$

which leads to $h = 2.7R - 0.7r \approx 2.7R$. With R = 14.0 cm, we have h = (2.7)(14.0 cm) = 37.8 cm. (**b**) The energy considerations shown above (now with h = 6R) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \tfrac{7}{10} v_{cm}^2 + gR,$$

which gives us $v_{cm}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q leads to

$$N = m\frac{v_{cm}^2}{R - r} = m\frac{50gR}{7(R - r)}$$

which (for $R >> r$) gives $N \approx (50/7)mg$ = (50/7)(2.80×10^{-4} kg)(9.80 m/s^2) = 1.96×10^{-2} N.

(**c**) The direction is toward the center of the loop.

15. (**a**) The derivation of the acceleration is found in §11-4; Eq. 11-13 gives

$$a_{cm} = -\frac{g}{1 + I_{cm}/MR_0^2}$$

where the positive direction is upward. We use I_{cm} = 950 g·cm^2, M = 120 g, R_0 = 0.320 cm and g = 980 cm/s^2 and obtain

$$|a_{cm}| = \frac{980}{1 + (950)/(120)(0.32)^2} = 12.5 \approx 13 \text{ (cm/s}^2).$$

(**b**) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{cm} = (½)a_{cm}t^2$. Thus, we set y_{cm} = –120 cm, and find

$$t = \sqrt{\frac{2y_{cm}}{a_{cm}}} = \sqrt{\frac{2(-120\text{ cm})}{-12.5\text{ cm/s}^2}} = 4.38 \text{ s} \approx 4.4 \text{ s}.$$

(**c**) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{cm} = a_{cm}t = (-12.5 \text{ cm/s2})(4.38 \text{ s}) = -54.8 \text{ cm/s} ,$$

so its linear speed then is approximately 55 cm/s.

(**d**) The translational kinetic energy is

$$\tfrac{1}{2} mv_{cm}^2 = \tfrac{1}{2}(0.120 \text{ kg})(0.548 \text{ m/s})^2 = 1.8\times10^{-2} \text{ J}.$$

(**e**) The angular velocity is given by $\omega = -v_{cm}/R_0$ & the rotational kinetic energy is

$$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}I_{cm}\frac{v_{cm}^2}{R_0^2} = \frac{1}{2}(9.50\times10^{-5})\frac{(0.548)^2}{(3.2\times10^{-3})^2},$$

which yields K_{rot} = 1.4 J. (**f**) The angular speed is

$$\omega = |v_{cm}|/R_0 = (0.548 \text{ m/s}) / (3.2\times10^{-3} \text{ m})$$
$$= 1.7\times10^2 \text{ rad/s} = 27 \text{ rev/s}.$$

21. If we write $\boldsymbol{r} = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then (using Eq. 3-30) we find $\boldsymbol{r}\times\boldsymbol{F}$ is equal to <*cf. Prob.11-96*>

$$(yF_z - zF_y)\,\hat{\mathrm{i}} + (zF_x - xF_z)\,\hat{\mathrm{j}} + (xF_y - yF_x)\,\hat{\mathrm{k}}.$$

(**a**) Plugging in, we find $\boldsymbol{\tau}$ = [(3.0 m)(6.0 N)–(4.0 m)(–8.0 N)]**k** = (50 kN·m)**k**. (**b**) We use Eq.3-27, $|\boldsymbol{r}\times\boldsymbol{F}| = rF\sin\phi$, where ϕ is the angle between $\boldsymbol{r}$ and $\boldsymbol{F}$.

Now $r = (x^2+y^2)^{1/2}$ = 5.0 m and $F = (F_x^2+F_y^2)^{1/2}$ = 10 N. Thus, rF = (5.0 m)(10 N) = 50 N·m, the same as the magnitude of the vector product calculated in part (a). This implies $\sin\phi = 1$ and $\phi = 90°$.

24. We note that the component of $\boldsymbol{v}$ perpendicular to $\boldsymbol{r}$ has magnitude $v\sin\phi$ where $\phi = 30°$. A similar observation applies to $\boldsymbol{F}$. (**a**) Eq. 11-20 leads to $\ell = rmv_\perp$ = (3.0)(2.0)(4.0)sin30° = 12 (kg·m²/s). (**b**) Using the right-hand rule for vector products, we find $\boldsymbol{r}\times\boldsymbol{p}$ points out of the page, or along the $+z$ axis, perpendicular to the plane of the figure. (**c**) Eq. 10-38 leads to $\tau = rF\sin\phi$ = (3.0) (2.0)sin30° = 3.0 (N·m). (**d**) Using the right-hand rule for vector products, we find $\boldsymbol{r}\times\boldsymbol{F}$ is also out of the page, or along the $+z$ axis, perpendicular to the plane of the figure.

31. If we write (for the general case) $\boldsymbol{r} = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then (using Eq. 3-30) we find $\boldsymbol{r}\times\boldsymbol{v}$ is equal to $(yv_z-zv_y)\mathbf{i}+(zv_x-xv_z)\mathbf{j}+(xv_y-yv_x)\mathbf{k}$. (**a**) The angular momentum is given by the vector product $\ell = m\boldsymbol{r}\times\boldsymbol{v}$, where $\boldsymbol{r}$ is the position vector of the particle, $\boldsymbol{v}$ is its velocity, and m = 3.0 kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)[(3.0)(-6.0) - (8.0\text{ m})(5.0\text{ N})]\,\hat{\text{k}}$$
$$= (-1.7\times10^2\text{ kg·m}^2/\text{s})\,\hat{\text{k}}\,.$$

(**b**) The torque is given by Eq. 11-14, $\boldsymbol{\tau} = \boldsymbol{r}\times\boldsymbol{F}$. We write $\boldsymbol{r} = x\mathbf{i}+y\mathbf{j}$ and $\boldsymbol{F} = F_x\mathbf{i}$ and obtain

$$\vec{\tau} = (x\,\hat{\text{i}} + y\,\hat{\text{j}})\times(F_x\,\hat{\text{i}}) = -yF_x\,\hat{\text{k}}\,.$$

Since $\hat{\text{i}}\times\hat{\text{i}} = 0$ and $\hat{\text{j}}\times\hat{\text{i}} = -\hat{\text{k}}$. Thus, we find

$$\vec{\tau} = -(8.0\text{ m})(-7.0\text{ N})]\,\hat{\text{k}} = (56\text{ N·m})\,\hat{\text{k}}\,.$$

(**c**) According to Newton's second law $\boldsymbol{\tau} = d\ell/dt$, so the rate of change of the angular momentum is 56 kg·m²/s², in the positive z direction.

37. (**a**) A particle contributes mr^2 to the rotational inertia, where r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + md^2 = 14md^2$$
$$= 14(2.3\times10^{-2}\text{ kg})(0.12\text{ m})^2 = 4.6\times10^{-3}\text{ kg·m}^2.$$

(**b**) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus

$$L_m = 4md^2\omega = 4(2.3\times10^{-2}\text{ kg})(0.12\text{ m})^2(0.85\text{ rad/s})$$
$$= 1.1\times10^{-3}\text{ kg·m}^2/\text{s}.$$

(**c**) The total angular momentum is

$$I\omega = 14md^2\omega = 14(2.3\times10^{-2}\text{ kg})(0.12\text{ m})^2$$
$$(0.85\text{ rad/s}) = 3.9\times10^{-3}\text{ kg·m}^2/\text{s}.$$

42. (**a**) We apply conservation of angular momentum: $I_1\omega_1 + I_2\omega_2 = (I_1+I_2)\omega$. The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1+I_2} = \frac{(3.3)(450)+(6.6)(900)}{3.3+6.6}$$
$$= 750\text{ (rev/min)}.$$

(**b**) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1+I_2} = \frac{(3.3)(450)+(6.6)(-900)}{3.3+6.6}$$
$$= -450\text{ (rev/min)},$$

or $|\omega|$ = 450 rev/min. (**c**) The minus sign indicates that $\boldsymbol{\omega}$ is in the direction of the second disk's initial angular velocity - clockwise.

51. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\phi$ where m = 0.003 kg and $\phi = 60°$. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is $I = I_{rod} + mr^2$, where $I_{rod} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv\sin\phi = (mr^2 + \tfrac{1}{12}ML^2)\omega\,.$$

Thus, with ω = 10 rad/s, we obtain

$$v = \frac{(0.003)(0.25)+(4.0)(0.5)^2/12}{(0.25)(0.003)\sin 60°}\,10 = 1.3\times10^3\text{ (m/s)}.$$

60. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}\,,$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation: $mvd = (I_{\text{rod}}+md^2)$, where I_{rod} is found using Table 10-2(e) & the parallel axis theorem: $(I_{\text{rod}}+md^2) = Md^2/12 + M(d/2)^2 = Md^2/3$. Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = md\sqrt{2gh}\,/(\tfrac{1}{3}Md^2 + md^2),$$

which means the system has kinetic energy (½)$(I_{\text{rod}} + md^2)\omega^2$, which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1-\cos\theta)$, so we have

$$\tfrac{1}{2}(I_{rod} + md^2)\omega^2 = mgH + Mg(\tfrac{1}{2})H$$
$$\Rightarrow\ \frac{1}{2}\frac{m^2d^2(2gh)}{(Md^2/3)+md^2} = (m+\frac{M}{2})gd(1-\cos\theta)\,,$$

from which we obtain

$$\theta = \cos^{-1}[1-\frac{m^2h/d}{(m+M/2)(m+M/3)}]$$
$$= \cos^{-1}[1-\frac{h/d}{(1+M/2m)(1+M/3m)}].$$

$$\theta = \cos^{-1}[1 - \frac{20/40}{(1+1)(1+2/3)}] = \cos^{-1}(0.85) = 32°.$$

64.• (**a**) We choose clockwise as the negative rotational sense and rightwards as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes $v_{cm} = -R\omega = (-0.11 \text{ m})\omega$. This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in Fig.11-57. (**b**) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{cm} leads to

$$a_{\text{cm}} = -\mu_k g = -(0.21)(9.80 \text{ m/s}^2) = -2.1 \text{ m/s}^2.$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating. (**c**) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu_k mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu_k mgR}{2mR^2/5} = \frac{-5\mu_k g}{2R} = \frac{-5(0.21)(9.80)}{2(0.11)}$$

$$= -47 \text{ (rad/s}^2),$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up"). (**d**) The center-of-mass of the sliding ball decelerates from $v_{\text{cm},0}$ to v_{cm} during time t according to Eq. 2-11: $v_{\text{cm}} = v_{\text{cm},0} - \mu_k gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha|\, t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R},$$

where we have made use of our part (a) result in the last equality. We have two eqs. involving v_{cm}, so we eliminate that variable and find

$$t = \frac{2v_{\text{cm},0}}{7\mu_k g} = \frac{2(8.5)}{7(0.21)(9.80)} = 1.2 \text{ (s)}.$$

(**e**) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{cm},0}t - \tfrac{1}{2}\mu_k g t^2$$

$$= (8.5)(1.2) - (½)(0.21)(9.80)(1.2)^2 = 8.6 \text{ (m)}.$$

(**f**) The center of mass velocity at the time found in part (d) is

$$v_{\text{cm}} = v_{\text{cm},0} - \mu_k gt = 8.5 - (0.21)(9.80)(1.2) = 6.1 \text{ (m/s)}.$$

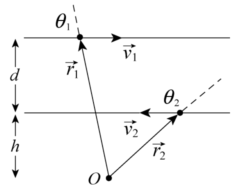
67.• (**a**) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1\sin\theta_1 = mv(d+h)$$

and it is into the page. The angular momentum of particle 2 has magnitude

$$\ell_2 = mvr_2\sin\theta_2 = mvh$$

and it is out of the page. The net angular momentum has magnitude



$$L = mv\,(d+h) - mv\,h = mv\,d$$

$= (2.90\times10^{-4} \text{ kg})(5.46 \text{ m/s})(0.042 \text{ m})$

$= 6.65\times10^{-5} \text{ kg·m}^2\text{/s},$

and is into the page. This result is independent of the location of the origin. (**b**) As indicated above, the expression does not change. (**c**) Suppose particle 2 is traveling to the right. Then

$$L = mv(d+h) + mv\,h = mv(d+2h).$$

This result depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$. (**d**) As we have seen in part (c), the result depends on the choice of origin.

72.• Conservation of energy (with Eq. 11-5) gives

(Mechanical Energy at max height up the ramp)
= (Mechanical Energy on the floor)

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I_{\text{cm}}\omega_f^2 + mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2,$$

where $v_f = \omega_f = 0$ at the point on the ramp where it (momentarily) stops. We note that the height h relates to the distance traveled along the ramp d by $h = d\sin(15°)$. Using item (*f*) in Table 10-2 and Eq. 11-2, we obtain

$$mgd\sin(15°) = mv^2(\tfrac{1}{2} + \tfrac{1}{5}).$$

After canceling m and plugging in $d = 1.5$ m, we find $v = 2.33$ m/s.

77.• The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I+MR^2)\omega$, which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground. (**a**) Angular momentum conservation leads to

$$0 = (I+MR^2)\omega - mR\,v \quad\Rightarrow\quad \omega = \frac{mRv}{I+MR^2}.$$

(**b**) The girl's linear speed is given by Eq. 10-18:

$$R\omega = \frac{mR^2v}{I+MR^2}.$$

81.• (**a**) Interpreting h as the height increase for the center of mass of the body, then (using Eq. 11-5) mechanical energy conservation leads to $K_i = U_f$,

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = mgh,\quad \frac{1}{2}mv^2 + \frac{1}{2}I(\frac{v}{R})^2 = mg(\frac{3v^2}{4g}),$$

from which v cancels and we obtain $I = (½)mR^2$. (**b**) From Table 10-2(c), we see that the body could be a solid cylinder.

Ex.5-2: Prob.11-85.

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

重點整理－第 11 章 滾動、力矩與角動量
對車輪之危險為何？

滾動體 對於半徑爲 R 的滾子不打滑(平穩)地滾動時，

$$v_{cm} = \omega R, \tag{11-2}$$

式中 v_{cm} 爲滾輪質心的線速度，而 ω 爲滾輪對於輪心的角速度，滾輪也可視爲某瞬間繞著滾輪與路面接觸點 P 轉動，滾輪繞此點轉動的角速度與對於輪心的相同，滾輪具有動能爲

$$K = \tfrac{1}{2} I_{cm}\omega^2 + \tfrac{1}{2} M v_{cm}^2, \tag{11-5}$$

其中 I_{cm} 爲滾輪對於質心的轉動慣量，而 M 爲輪子質量。假如滾輪加速但依然平穩地滾動，則質心加速度 a_{cm} 與對於中心的角加速度 α 的關係爲

$$a_{cm} = \alpha R. \tag{11-6}$$

假如滾輪平穩地從傾角 θ 的斜面滾下，它於沿著斜面 x 軸的加速度爲

$$a_{cm,x} = -g \sin\theta / (1 + \frac{I_{com}}{MR^2}). \tag{11-10}$$

力矩為向量 於三維中力矩 $\vec{\tau}$ 爲向量，其相對於固定點(通常爲原點)定義爲

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{11-14}$$

or $(yF_z - zF_y)\hat{\text{i}} + (zF_x - xF_z)\hat{\text{j}} + (xF_y - yF_x)\hat{\text{k}}$. (分量式)

式中 $\vec{F}$ 爲施於質點的力，而 $\vec{r}$ 爲相對於固定點定位質點的位置向量。力矩 $\vec{\tau}$ 的大小爲

$$\tau = rF\sin\phi = r F_\perp = r_\perp F, \tag{11-15,16,17}$$

式中 ϕ 爲 $\vec{F}$ 與 $\vec{r}$ 的夾角，$F_\perp$ 爲 $\vec{F}$ 垂直 $\vec{r}$ 的分量，而 $r_\perp$ 爲 $\vec{F}$ 的力臂。$\vec{\tau}$ 的方向則由右手定則給定。

質點的角動量 具線動量 $\vec{p}$、質量 m 及線速度 $\vec{v}$ 的質點之角動量 $\vec{\ell}$ 爲一向量，其相對於固定點(通常爲原點)定義爲

$$\vec{\ell} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v}) \tag{11-18}$$

or $(yp_z - zp_y)\hat{\text{i}} + (zp_x - xp_z)\hat{\text{j}} + (xp_y - yp_x)\hat{\text{k}}$. (分量式)

$\vec{\ell}$ 的大小爲 (11-19,20,21)

$$\ell = mv\sin\phi = rP_\perp = mrv_\perp = r_\perp p = r_\perp mv,$$

其中 ϕ 爲 $\vec{r}$ 與 $\vec{p}$ 的夾角，$P_\perp$ 與 $v_\perp$ 爲 $\vec{p}$ 及 $\vec{v}$ 分別與 $\vec{r}$ 垂直之分量，而 $r_\perp$ 爲固定點與 $\vec{p}$ 的延伸線的垂直距離，方向則由**右手定則**給定。

angular momentum 角動量; rolling 滾動; rolling without slipping 無滑動之滾動; smooth rolling 平穩滾動; gyroscope 陀螺儀; spinning 旋轉的; precession 進動; Ferris wheel 摩天輪; penguin 企鵝; ramp 斜坡; spacecraft 太空船; springboad 跳板; shock wave 震波; supersonic 超音速的; Segway 塞格威(電動滑板車/自動平衡車);

角式牛頓第二定律 對於質點以角式表示牛頓第二定律爲

$$\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt}, \tag{11-23}$$

式中 $\vec{\tau}_{net}$ 爲作用於質點上的淨力矩，而 $\vec{\ell}$ 爲質點的角動量。

質點系統的角動量 質點系統的角動量 $\vec{L}$ 爲各質點的角動量之向量和：

$$\vec{L} = \vec{\ell}_1 + \vec{\ell}_2 + \cdots + \vec{\ell}_n = \sum_{i=1}^{n} \vec{\ell}_i. \tag{11-26}$$

角動量的時變率等於作用於系統的淨外力矩（力矩的向量和乃由於系統內質點與系統外質點間交互作用所產生的）。

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}. \tag{11-29}$$

剛體的角動量 對於繞定軸轉動的剛體，角動量平行於轉軸的分量爲

$$L = I\omega \quad \text{(剛體, 固定軸)}. \tag{11-31}$$

角動量守恆 假如作用於系統的淨外力矩爲零時，系統的角動量 $\vec{L}$ 維持不變

$$\vec{L} = \text{常數} \quad \text{(孤立系統)} \tag{11-32}$$

或

$$\vec{L}_i = \vec{L}_f \quad \text{(孤立系統)} \tag{11-33}$$

這即**角動量守恆定律**。它是自然界基本的守恆定律之一，甚至於牛頓第二運動定律不適用的狀況，它亦被證實。

迴轉儀的進動 自轉的陀螺儀(迴轉儀)可繞著通過其支座的垂直軸進動，其速率爲

$$\Omega = \frac{Mgr}{I\omega}, \tag{11-46}$$

式中 M 爲陀螺儀的質量，r 爲力臂，I 爲轉動慣量，而 ω 爲自轉(角)速率。

Yo-Yo 溜溜球; skateboard 滑板; in-line skates 直排輪;

J1. "溜溜球的弦理論" 【科技解剖室】，菲謝蒂，科學人 No.32 (2004 年 10 月號) 114。 J2 "陀螺儀—隱形嚮導" 【科技解剖室】，菲謝蒂，科學人 No.6 (2002 年 8 月號) 116。 J3. "處處好運轉"【科技解剖室】，菲謝蒂，科學人 No. 43 (2005 年 9 月號) 102-103。 J4. "風的故事—從風車到風力機"，呂威賢，科學發展 383 期 (9311) 6。 J5. "飛機的眼睛－淺談陀螺儀"，劉得彰，科學月刊，17 卷 9 期，民 75 年。

•塞格威網站 http://www.segway.tw

Prob. *12- 5, 7, 12, 19, 29, 33, 39, 40, 43, 48, 53, 54, 70 (tentatively)*

•備忘錄•