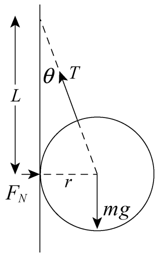
Chapter ***12*** ***Equilibrium and Elasticity***

05. Three forces act on the sphere: the tension force $\boldsymbol{T}$ of the rope (acting along the rope), the force of the wall $\boldsymbol{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\boldsymbol{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then, the vertical component of Newton's second law is $T\cos\theta - mg = 0$. The horizontal component is $F_N - T\sin\theta = 0$. (**a**) We solve the first eq. for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain



$$T = mg\frac{\sqrt{L^2+r^2}}{L} = 9.4\ \text{(N)}$$

$$= (0.85)(9.8)\frac{\sqrt{0.080^2+0.042^2}}{0.080}.$$

(**b**) We solve the second eq. for the normal force: $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}}.$$

$$= \frac{mgr}{L} = \frac{(0.85)(9.8)(0.042)}{0.080} = 4.4\ \text{(N)}$$

07. We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. W is the weight of the diver, located at $x = L$. The following two eqs. result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0 \quad \text{and} \quad F_1 d + W(L-d) = 0.$$

(**a**) The second eq. gives

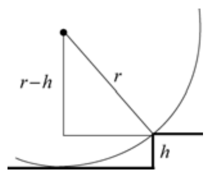
$$F_1 = -\frac{L-d}{d}W = -\frac{3.0\ \text{m}}{1.5\ \text{m}}(580\ \text{N}) = -1160\ \text{N},$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N. (**b**) Since F_1 is negative, indicating that this force is downward. (**c**) The first eq. gives $F_2 = W - F_1 = 580\ \text{N} + 1160\ \text{N} = 1740$ N, which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N. (**d**) The result is positive, indicating that this force is upward. (**e**) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched. (**f**) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

12. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude $mg = 500$ N, is a horizontal distance $\ell = 0.375$ m from C. The applied force of magnitude $F = 350$ N is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\ \text{N})(0.375\ \text{m})}{350\ \text{N}} = 0.536\ \text{m}.$$

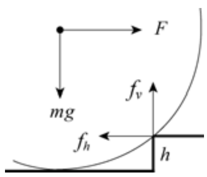
19. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force $\boldsymbol{F}$ applied horizontally at the axle, the force of gravity $m\boldsymbol{g}$ acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero. We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step. The distance from the line of mg to the corner is $[r^2+(r-h)^2]^{1/2} = (2rh-h^2)^{1/2}$. Thus,



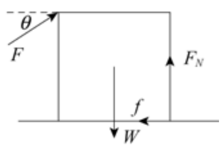
$$F(r-h) - mg\sqrt{2rh-h^2} = 0.$$

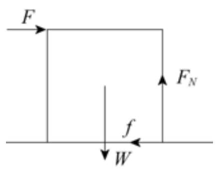
The solution for F is

$$F = \frac{\sqrt{2rh-h^2}}{r-h}mg = 13.6\ \text{N}.$$



29. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram below. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes: $FL - WL/2 = 0$. Here L is the length of a side of the box and the origin was chosen to be at the lower right edge. (**a**) From the torque eq., we find $F = W/2 = 890\ \text{N}/2 = 445$ N. (**b**) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the eqs. of equilibrium $F_N = W = 890$N and $f = F = 445$N, so $\mu_s = 445\text{N}/890\text{N} = 0.50$. (**c**) The box can be rolled with a smaller applied force if the force points

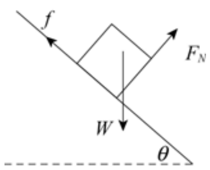




upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque eq. then becomes $FL\cos\theta + FL\sin\theta - WL/2 = 0$, with the solution $F = W/2(\cos\theta+\sin\theta)$. We want $\cos\theta+\sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45°} = \frac{890\,\text{N}}{4\cos 45°} = 315\,\text{N}.$$

33. The force diagram shown below depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, F_N is the normal force of the plane on the crate, and f is the force of friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(**a**) The x and y components of Newton's second law are

$$W\sin\theta - f = 0 \text{ and } F_N - W\cos\theta = 0.$$

respectively. The y eq. gives $F_N = W\cos\theta$. Since the crate is about to slide

$$f = \mu_s F_N = \mu_s W\cos\theta,$$

where μ_s is the coefficient of static friction. We substitute into the x eq. and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \quad\Rightarrow\quad \tan\theta = \mu_s.$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1}0.60 = 31.0°$. In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude $f\,h$, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $F_N\ell/2$, where ℓ is the length of an edge. Since the total torque vanishes, $f\,h = F_N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $F_N = W\cos\theta$. Substituting these expressions into the torque eq., we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7°.$$

As θ is increased from zero the crate slides before it tips. (**b**) It starts to slide when $\theta = 31°$. (**c**) The crate begins to slide when $\theta = \tan^{-1}\mu_s = \tan^{-1}0.70 = 35.0°$ and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased. (**d**) Tipping begins at $\theta = 33.7° \approx 34°$.

39. (**a**) Let $\boldsymbol{F}_A$ and $\boldsymbol{F}_B$ be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A/AE$, where A is the cross-sectional area of the wire and E is Young's modulus for steel (200×10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B/AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell.$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross–sectional area of a wire is $A = \pi r^2 = (1.20\times10^{-3}\text{ m})^2 = 4.52\times10^{-6}\text{ m}^2$. Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$F_A = \frac{(103)(9.8)(2.50) + (4.52\times10^{-6})(200\times10^9)(2.0\times10^{-3})}{2.50 + 2.50}$$

$$= 866\text{ (N)}.$$

(**b**) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_B = (103\text{ kg})(9.8\text{ m/s}^2) - 866\text{ N} = 143\text{ N}.$$

(**c**) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque eq. becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$

40. (**a**) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{FA}{A_A E_A} \quad\text{and}\quad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B},$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)(2E_B)}{A_B E_B} = 4.$$

When we combine this ratio with the eq. $F_A + F_B = W$, we find $F_A/W = 4/5 = 0.80$. (**b**) This also leads to the result $F_B/W = 1/5 = 0.20$. (**c**) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$ which leads to

$$d_A / d_B = F_B / F_A = 1/4 = 0.25\,.$$

43. With the x axis parallel to the incline (positive uphill), then

$$\Sigma F_x = 0 \Rightarrow T\cos 25° - mg\sin 45° = 0,$$

where reference to Fig. 5-18 in the textbook is helpful. Therefore, $T = 76$ N.

53. (**a**) The center of mass of the top brick cannot be further (to the right) with respect to the brick below it (brick 2) than (½)L; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = (½)L$ in the maximum case. (**b**) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = (¼)L$. (**c**) Now the total center of mass of bricks 1, 2 and 3 is one-third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{cm}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6},$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$. (**d**) A similar calculation

$$x'_{\text{cm}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$. (**e**) We find $h = \Sigma_{i=1} a_i = 25L/24$.

70. The notation and coordinates are as shown in Fig. 12-6 in the textbook. Here, the ladder's center of mass is halfway up the ladder (unlike in the textbook figure). Also, we label the x and y forces at the ground f_s and F_N, respectively. Now, balancing forces, we have

$$\Sigma F_x = 0 \Rightarrow f_s = F_w, \quad \Sigma F_y = 0 \Rightarrow F_N = mg.$$

Since $f_s = f_{s,\max}$, we divide the eqs. to obtain

$$\frac{f_{s,\max}}{F_N} = \mu_s = \frac{F_w}{mg}.$$

Now, from $\Sigma\tau_z = 0$ (with axis at the ground) we have $mg(a/2) - F_w h = 0$. But from the Pythagorean theorem, $h = \sqrt{L^2 - a^2}$, where L = length of ladder. Therefore,

$$\frac{F_w}{mg} = \frac{a/2}{h} = \frac{a}{2\sqrt{L^2 - a^2}}.$$

In this way, we find

$$\mu_s = \frac{a}{2\sqrt{L^2 - a^2}} \quad \Rightarrow \quad a = \frac{2\mu_s L}{\sqrt{1 + 4\mu_s^2}}.$$

Therefore, $a = 3.4$ m.

重點整理－第 12 章 平衡與彈性

浮腫、疼痛與攀岩間關聯為何？ **平衡條件**：(1)**合(外)力須為零** $\vec{F}_{net} = 0$ **及**(2)**合(外)力矩須為零** $\vec{\tau}_{net} = 0$；**重力產生的總力矩正如總重力作用於重(質)心上。**

靜力平衡 靜止的剛體稱其處於靜力平衡。對於此類的物體，作用於其上外力之向量和爲零：

$$\vec{F}_{net} = 0 \qquad (\text{力平衡}). \quad (12\text{-}3)$$

假如所有力皆位於 xy 平面，上述的向量式等同兩個分量式 $F_{\text{net},x} = 0$ 及 $F_{\text{net},y} = 0$ (力平衡). (12-7,8)

靜力平衡亦意謂對任意點作用於物體的外力矩之向量和爲零

$$\vec{\tau}_{net} = 0 \qquad (\text{力矩平衡}). \quad (12\text{-}5)$$

假如所有力位於 xy 平面，則所有力矩向量皆與 z 軸平行， 於是 12-5 式等同單一分量式

$$\tau_{\text{net},z} = 0 \qquad (\text{力矩平衡}). \quad (12\text{-}9)$$

重心 重力各別作用於組成物體的各元素上，所有各別作用的淨效應時可藉想像一等效總重力 $\vec{F}_g$ 作用於重心上而求出；假如物體的所有元素之重力加速度 $\vec{g}$ 均相同，則重心位於質心上。

彈性模數 當物體對於作用於其上之力反應時，三種彈性模數用以描述物體之彈性行爲(形變)。應變(**長度改變的比率**)依據下列的一般關係，藉適當的係數與應力(**單位面積所受的力**)成線性相關，

$$\text{應力} = \text{模數} \times \text{應變}. \qquad (12\text{-}22)$$

伸張與壓縮 當物體在伸張或壓縮狀況下，12-22 式寫成

$$\frac{F}{A} = E\frac{\Delta L}{L}, \qquad (12\text{-}23)$$

其中$\Delta L/L$ 爲物體的張應變或壓應變，F 爲造成此應變的施力 $\vec{F}$ 大小，A 爲 $\vec{F}$ 作用的截面積(垂直 A，如圖 12-11a)，而 E 爲物體的楊氏模數；應力爲 F/A。

切應力與切應變 當物體處於切應力狀況下，12-22 式寫成

$$\frac{F}{A} = G\frac{\Delta x}{L}, \qquad (12\text{-}24)$$

其中$\Delta x/L$ 爲物體的切應變，Δx 爲沿著施力方向物體末端的位移（如圖 12-11b），而 G 爲物體的切變模數；應力爲 F/A。

液壓應力 當物體由於週遭流體所施應力而使其處於液壓壓縮狀況下，12-22 式寫成

$$p = B\frac{\Delta V}{V}, \qquad (12\text{-}25)$$

其中 p 爲流體作用於物體上的壓力(液壓應力)，$\Delta V/V$ (應變)爲壓力造成的物體體積改變率之絕對值，而 B 爲物體之體彈性模數。

48. (**a**) Eq. 12-8 leads to $T_1 \sin 40^\circ + T_2 \sin\theta = mg$. Also, Eq. 12-7 leads to $T_1 \cos 40^\circ - T_2 \cos\theta = 0$. Combining these gives the expression

$$T_2 = \frac{mg}{\cos\theta \tan 40^\circ + \sin\theta}.$$

To minimize this, we can plot it or set its derivative equal to zero. In either case, we find that it is at its minimum at $\theta = 50°$. (**b**) At $\theta = 50°$, we find $T_2 = 0.77mg$.

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

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(static) equilibrium (靜力)平衡; center of gravity 重心; (modulus of) elasticity 彈性(模數); tension 伸張; tensile 伸張的; compression 壓縮; strain 應變; strain gauge 應變規; stress 應力; hydraulic 液壓的; yield/ultimate strength 屈服/極限強度; shearing 受切變; Young's/shear/bulk modulus 楊式/切變/體模數; beam 橫樑; bulge 浮腫; ladder 梯子; lattice 晶格; polystyrene 聚苯乙烯;

•**備忘錄**•

54.* (**a**) With $F = ma = -\mu_k mg$ the magnitude of the deceleration is

$$|a| = \mu_k g = (0.40)(9.8 \text{ m/s}^2) = 3.92 \text{ m/s}^2.$$

(**b**) As hinted in the problem statement, we can use Eq. 12-9, evaluating the torques about the car's center of mass, and bearing in mind that the friction forces are acting horizontally at the bottom of the wheels; the total friction force there is $f_k = \mu_k mg = 3.92m$ (with SI units understood – and m is the car's mass), a vertical distance of 0.75 meter below the center of mass. Thus, torque equilibrium leads to

$$(3.92m)(0.75) + F_{N,r}(2.4) - F_{N,f}(1.8) = 0.$$

Eq. 12-8 also holds (the acceleration is horizontal, not vertical), so we have $F_{N,r} + F_{N,f} = mg$, which we can solve simultaneously with the above torque eq.. The mass is obtained from the car's weight: $m = 11000/9.8$, and we obtain $F_{N,r} = 3929 \approx 4000$ N. Since each involves <u>two</u> wheels then we have (roughly) 2.0×10^3 N on each rear wheel. (**c**) From the above eq., we also have $F_{N,f} = 7071 \approx 7000$ N, or 3.5×10^3 N on each front wheel, as the values of the individual normal forces. (**d**) Eq. 6-2 directly yields (approximately) 7.9×10^2 N of friction on each rear wheel, (**e**) Similarly, Eq. 6-2 yields 1.4×10^3 N on each front wheel.