Chapter 15 Oscillations

01. (**a**) The amplitude is half the range of the displacement, or x_m = 1.0 mm. (**b**) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120\ \text{Hz})(1.0\times10^{-3}\ \text{m}) = 0.75\ \text{m/s}.$$

(**c**) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m$$
$$= [2\pi(120\ \text{Hz})]^2(1.0\times10^{-3}\ \text{m}) = 5.7\times10^2\ \text{m/s}^2.$$

05. (**a**) The motion repeats every 0.500 s so the period must be T = 0.500 s. (**b**) The frequency is the reciprocal of the period: $f = 1/T$ = 1/(0.500 s) = 2.00 Hz. (**c**) The angular frequency ω is $\omega = 2\pi f = 2\pi$ (2.00 Hz) = 12.6 rad/s. (**d**) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k: $k = m\omega^2$ = (0.500 kg) (12.6 rad/s)^2 = 79.0 N/m. (**e**) Let x_m be the amplitude. The maximum speed is $v_m = \omega x_m$ = (12.6 rad/s) (0.350 m) = 4.40 m/s. (**f**) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m$ = (79.0 N/m) (0.350 m) = 27.6 N.

14. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x = d/2$ and ω = 0.503 rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = x_m/2 = d/4$.

$$x = x_m\cos(\omega t+\phi),\quad d/4 = (d/2)\cos(0.503t+0),$$
$$1/2 = \cos(0.503t),$$

which has t = 2.08 h as the smallest positive root. The calculator is in radians mode during this calculation.

15. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by $F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.80)}{(2\pi\times2.0)^2}\,0.031\ (\text{m}).$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

20. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq.15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to g = 9.8 m/s^2. (**a**) Using Eq. 15-5 and T = 1.0 s, we have

$$(2\pi/T)^2 x_m = g \Rightarrow x_m = gT^2/4\pi^2 = 0.25\ \text{m}.$$

(**b**) Since $\omega = 2\pi f$, and x_m = 0.050 m is given, we find

$$(2\pi f)^2 x_m = g \quad\Rightarrow\quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2\ \text{Hz..}$$

24. Let the spring constants be k_1 and k_2. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x+k_2x|$ acting in a direction so as to return the block to its equilibrium position (x = 0). Since the acceleration $a = d^2x/d^2t$, Newton's second law yields

$$m\,(d^2x/dt^2) = -k_1x - k_2x\ .$$

Substituting $x = x_m\cos(\omega t+\phi)$ and simplifying, we find

$$\omega^2 = \frac{1}{m}(k_1+k_2),$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{m}(k_1+k_2)}\ .$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 30\ \text{Hz} \quad\text{and}\quad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 45\ \text{Hz},$$

respectively. Comparing these expressions, it is clear that

$$f = \sqrt{f_1^2 + f_2^2} = \sqrt{(30)^2 + (45)^2} = 54\ (\text{Hz}).$$

26. In order to find the effective spring constant for the combination of springs shown in Fig. 15-35, we do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\Delta x_\ell$. Thus $k_{\rm eff} = k\Delta x_\ell/2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace $k_{\rm eff}$ in $f = (1/2\pi)(k_{\rm eff}/m)^{1/2}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}\,.$$

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.

31. The total energy is given by $E = (½)kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first. (**a**) The fraction of the energy that is kinetic is

$$\frac{K}{E} = \frac{E-U}{E} = 1-\frac{U}{E} = 1-\frac{1}{4} = 0.75,$$

where the result from part (b) has been used. (**b**) When $x = x_m/2$ the potential energy is $U = (½)kx^2 = (1/8)kx_m^2$. The ratio is

$$\frac{U}{E} = \frac{kx_m^2/8}{kx_m^2/2} = \frac{1}{4} = 0.25\,.$$

(**c**) Since $E = (½)kx_m^2$ and $U = (½)kx^2$, $U/E = x^2/x_m^2$. We solve $x^2/x_m^2 = 1/2$ for x. We should obtain $x = x_m/2^{1/2}$.

37. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m+M$ attached to a spring of spring constant k). (**a**) Momentum conservation readily yields $v' = mv/(m+M)$. With $m = 9.5$ g, $M = 5.4$ kg and $v = 630$ m/s, we obtain $v' = 1.1$ m/s. (**b**) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\tfrac{1}{2}(m+M)v'^2 = \tfrac{1}{2}kx_m^2 \Rightarrow \tfrac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \tfrac{1}{2}kx_m^2,$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3})(630)}{\sqrt{(6000)(9.5\times10^{-3}+5.4)}} = 3.3\times10^{-2}\ \text{(m)}.$$

43. (**a**) A uniform disk pivoted at its center has a rotational inertia of $(½)Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r+L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $(½)Mr^2+M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$I = \tfrac{1}{2}Mr^2 + M(L+r)^2 + \tfrac{1}{2}mL^2 = \tfrac{1}{2}(0.500)(0.100)^2 + (0.500)(0.500+0.100)^2 + \tfrac{1}{2}(0.270)(0.500)^2 = 0.205\ (\text{kg}\cdot\text{m}^2).$$

(**b**) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\text{ m} + 0.100\text{ m} = 0.600\text{ m},$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\text{ m})/2 = 0.250$ m away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{M\ell + m\ell}{M+m} = 0.477\ \text{(m)} = \frac{(0.500)(0.600)+(0.270)(0.250)}{0.500+0.270}\,.$$

(**c**) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 1.50\ \text{(s)} = 2\pi\sqrt{\frac{0.205}{(0.500+0.270)(9.80)(0.447)}}\,.$$

76.• (**a**) The problem gives the frequency $f = 440$ Hz, which means a cycle-per-second. The unit of angular frequency ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2.8\times10^3$ rad/s. (**b**) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With $x_m = 0.00075$ m and the above value for ω, this expression yields $v_m = 2.1$ m/s. (**c**) In the discussion immediately after Eq. 15-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value $\omega = 2765$ rad/s is used) yields $a_m = 5.7$ km/s^2.

83.• (**a**) We use Eq.15-29 and the parallel-axis theorem $I = I_{\rm cm}+mh^2$ where $h = R = 0.126$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{\rm cm} = (½)mR^2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2 + mR^2/2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.873\ \text{(s)}.$$

(**b**) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2+2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{1}{4}[\,3R \pm \sqrt{(3R)^2 - 8R^2}\,] = R \text{ or } \frac{1}{2}R.$$

Thus, our result is $r = 0.126/2 = 0.0630$ m.

95.*AP* (**a**) We require $U = E/2$ at some value of x. Using Eq. 15-21, this becomes

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}(\tfrac{1}{2}kx_m^2) \quad\Rightarrow\quad x = \tfrac{1}{\sqrt{2}}x_m.$$

We compare the given expression x as a function of t with Eq. 15-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/2^{1/2} \approx 3.5$ (m). (**b**) We solve the given expression (with $x = 5.0/2^{1/2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ (s)}.$$

Since we are asked for the interval $t_{eq} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ (s)}.$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

96.• (**a**) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\tfrac{1}{2}kx_m^2 = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I_{cm}\omega^2 = \tfrac{1}{2}Mv_{cm}^2$$
$$+\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{4}Mv_{cm}^2 = \frac{3}{4}Mv_{cm}^2,$$

which leads to $Mv_{cm}^2 = (⅔)kx_m^2 = 0.125$J. The translational kinetic energy is therefore $(½)Mv_{cm}^2 = (⅓)kx_m^2 = 0.0625$J. (**b**) And the rotational kinetic energy is $(¼)Mv_{cm}^2 = kx_m^2/6 = 0.03125$ J $= 3.13\times 10^{-2}$ J. (**c**) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{3}{4}Mv_{cm}^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right)$$
$$= \frac{3}{2}Mv_{cm}a_{cm} + kxv_{cm} = 0,$$

which leads to $a_{cm} = -(⅔)(k/M)x$. Comparing with Eq. 15-8, we see that $\omega = (2k/3M)^{1/2}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi(3M/2k)^{1/2}$.

重點整理－第 15 章 振盪
訓練有素跳水選手之高彈跳秘密為何？

♦**振動** 質點(物體)作來回往復運動；

oscillation 振盪; vibration 振動; oscillator 振子; resonance 共振; (simple) pendulum(單)擺; physical pendulum 物理擺; bob 吊錘; suspension point 懸點; fulcrum 支點; torsion pendulum/constant 扭擺/扭力常數; simple harmonic motion (SHM)簡諧運動; restoring force/torque 回復力/力矩; cycle 循環; period 週期; (angular) frequency (角)頻率, amplitude 振幅; phase constant/angle 相位常數/角; phase 相位; hertz (Hz)赫; damping force/constant 阻尼力/常數; forced/driven oscillation 強迫/驅駛振盪; hallmark 特徵; •**備忘錄**•

♦**週期運動** 振動系統(物體)一而再地重覆相同運動或任何運動有規律地在一定時間距內重現！①**週期** T：完成一完整振動所需的時間，單位：second (s)；②**頻率** f：單位時間完成振動循環之次數，單位：Hz ($\equiv s^{-1}$)；③**振幅** x_m(> 0)：物體偏離平衡位置之最大位移量。*Note* $f = 1/T$；

簡諧運動(SHM) 質點之位移與時間的關係為諧和函數(**S**: Simple, 振幅 x_m = const, **H**: Harmonic),

$$x(t) = x_m\cos(\omega t+\phi) \text{ or } x(t) = x_m\sin(\omega t+\phi)$$

$x(t)$：位移，x_m：振幅，$\omega t+\phi$：相位，

角頻率 ω (in rad/s)：$\omega T = 2\pi \Rightarrow \omega = 2\pi/T = 2\pi f$

速度 $v(t) = dx/dt = -\omega x_m\sin(\omega t+\phi)$，

♦速度振幅(速率最大值) $v_m = \omega x_m$，

加速度 $a(t) = dv/dt = -\omega^2 x_m\sin(\omega t+\phi)$，

♦加速度振幅 $a_m = \omega^2 x_m$； $a(t) = -\omega^2 x(t)$，

(1)加速度大小正比於物體偏離平衡之位移大小且(2)兩者永遠反向。

簡諧運動之作用力

$$F(t) = ma(t) = -m\omega^2 x(t) \text{ —回復力，}$$

木塊(m)-彈簧(k)系統 $\omega^2 = k/m$ or $\omega = \sqrt{k/m}$ ，

$f = \omega(2\pi)^{-1} = (2\pi)^{-1}\sqrt{k/m}$ ，$T = 1/f = 2\pi\sqrt{m/k}$ ，

簡諧運動之能量：系統無耗能機制時，

力學能 E = 動能 K + 位能 U = 常數。

系統之**動能**及**位能**持續交換，但其總和仍不變！

木塊(m)及彈簧(k)系統

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 = \text{cont.}$$

單擺：一理想化模型，質點由無質量之弦懸掛。小角度($|\theta| \ll 1$)擺動為**簡諧運動**：**角頻率** ω：$\omega^2 = g/L$，**頻率** $f = \omega/(2\pi)$ or $f = (2\pi)^{-1}\sqrt{g/L}$ ，

週期 $T = 2\pi\sqrt{L/g}$ ，只與擺長 L 及重力加速度 g 有關，而與質量 m 無關。♠擺長 $L = 1.00$ m $\Rightarrow T = 2.007$ s；$L = 2.45$ m $\Rightarrow T = 3.14$ s ($g = 9.80$ m/s^2)。

♦弦與鉛直線之夾角 $\theta(t) = \theta_m\cos(\sqrt{g/L}\,t+\phi)$.

Note 單擺以有限擺角 θ_M 擺動時，其週期

$$T = 2\pi\sqrt{L/g}\,[1 + \tfrac{1}{4}\sin^2(\tfrac{1}{2}\theta_M) + \ldots]$$

物理擺：真實的擺，小角度($|\theta| \ll 1$)擺動為**簡諧運動**，$\omega^2 = mgh/I$，$f = \omega/2\pi = (1/2\pi)(mgh/I)^{½}$ or $T = 2\pi(I/mgh)^{½}$，I = 轉動慣量，h = 重心至懸點之距。

49. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = (C/I)^{1/2}$ and the period is $T = 2\pi/\omega = 2\pi(I/C)^{1/2}$, where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(½)L\sin\theta$ further from the wall and has moved a distance $(½)L(1-\cos\theta)$ to the right. The length of the spring is now $\{ (L/2)^2(1-\cos\theta)^2 + [\ell_0+(L/2)\sin\theta]^2 \}^{1/2}$. If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell_0+(½)L\theta$ and its elongation is $\Delta x = (½)L\theta$. The force it exerts on the rod has magnitude $F = k\Delta x = (½)kL\theta$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -F(½)L$, where the pivot point was taken as the origin. Thus $\tau = -(¼)(kL^2)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = (¼)kL^2$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}\,.$$

With m = 0.600 kg and k = 1850 N/m, we obtain T = 0.0653 s. (*cf. S.P. 15-6*)

圓柱體(截面積 A)浮於水面，平衡時沒入水中之深度 h，稍微鉛直下壓後放手，***Q***.圓柱體作SHM？U 型管內裝液體(液柱總長度為ℓ)，當液面些微擾動時，作 SHM？某半徑為 R 之均質圓球以一細弦懸掛著，試證明當圓球小角度擺動時作 SHM，並求其週期。設細弦質量可忽略及懸點至球心之距為 L。**●備忘錄●**