

Chapter 17

04. The density of oxygen gas is $\rho = M/V = 0.0320 \text{ kg} / 0.0224 \text{ m}^3 = 1.43 \text{ kg/m}^3$. From $v = (B/\rho)^{1/2}$ we find

$$B = v^2 \rho = (317 \text{ m/s})^2 (1.43 \text{ kg/m}^3) = 1.44 \times 10^5 \text{ Pa}.$$

05. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives $d = (1/2)gt_f^2$, or $t_f = (2d/g)^{1/2}$. The sound travels at a constant speed v_s , so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = (2d/g)^{1/2} + d/v_s$. This equation is to be solved for d . Rewrite it as $(2d/g)^{1/2} = t - d/v_s$, and square both sides to obtain

$$2d/g = t^2 - 2(t/v_s)d + d^2/v_s^2.$$

Now multiply by $g v_s^2$ and rearrange to get

$$g d^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for d . Its solutions are

$$d = \frac{1}{2g} [2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2v_s^2t^2}].$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7 \text{ m}$ is obtained.

08. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: $p_m = 1.50 \text{ Pa}$. **(b)** We identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi = 158 \text{ Hz}$. **(c)** We also obtain $\lambda = 2\pi/k = 2.22 \text{ m}$. **(d)** The speed of the wave is $v = \omega/k = 350 \text{ m/s}$.

16. At the location of the detector, the phase difference between the wave which traveled straight down the tube and the other one which took the semi-circular detour is

$$\Delta\phi = k\Delta d = (2\pi/\lambda)(\pi r - 2r).$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus

$$r_{\min} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0 \text{ cm}}{2(\pi - 2)} = 17.5 \text{ cm}.$$

23. The intensity is given by $I = (1/2)\rho v \omega^2 s_m^2$, where ρ is the density of air, v is the speed of sound in air, ω is the angular frequency, and s_m is the displacement amplitude for the sound wave. Replace ω with $2\pi f$ and solve for s_m : $s_m = 3.68 \times 10^{-8} \text{ (m)}$.

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00 \times 10^{-6}}{2\pi^2 (1.21)(343)(300^2)}}$$

26. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r = 3.00 \text{ m}$,

Waves – II

$I = 1.00 \times 10^{-6} / 4\pi(3.00)^2 = 8.84 \times 10^{-9} \text{ (W/m}^2\text{)}$. **(b)** The sound level there is

$$\beta = 10 \log \frac{8.84 \times 10^{-9}}{1.00 \times 10^{-12}} = 39.5 \text{ (dB)}.$$

30. (a) The intensity is $I = P_s/4\pi r^2 = 30.0 \text{ W}/4\pi(200 \text{ m})^2 = 5.97 \times 10^{-5} \text{ W/m}^2$. **(b)** Let $A (= 0.750 \text{ cm}^2)$ be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P = IA = (5.97 \times 10^{-5} \text{ W/m}^2)(0.750 \times 10^{-4} \text{ m}^2) = 4.48 \times 10^{-9} \text{ W}.$$

38. The frequency is $f = 686 \text{ Hz}$ and the speed of sound is $v_{\text{sound}} = 343 \text{ m/s}$. If L is the length of the air-column, then using Eq. 17-41, the water height is (in unit of meters) $h = 1.00 - L$

$$= 1.00 - \frac{nv}{4f} = 1.00 - \frac{n(343)}{4(686)} = 1.00 - 0.125n,$$

where $n = 1, 3, 5, \dots$ with only one end closed. **(a)** There are 4 values of n ($n = 1, 3, 5, 7$) which satisfies $h > 0$. **(b)** The smallest water height for resonance to occur corresponds to $n = 7$ with $h = 0.125 \text{ m}$. **(c)** The second smallest water height corresponds to $n = 5$ with $h = 0.375 \text{ m}$.

42. (a) Using Eq. 17-39 with $n = 1$ (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343 \text{ m/s}}{4(1.20 \text{ m})} = 71.5 \text{ Hz}.$$

(b) For the wire (using Eq. 17-53) we have

$$f' = n \frac{v_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}} \sqrt{\frac{\tau}{\mu}},$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$, both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension τ .

$$\tau = (2L_{\text{wire}}f')^2(m_{\text{wire}}/L_{\text{wire}}) = 4f'^2 m_{\text{wire}} L_{\text{wire}} = 4(71.5 \text{ Hz})^2(9.60 \times 10^{-3} \text{ kg})(0.330 \text{ m}) = 64.8 \text{ N}.$$

43. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f = nv/2L$, where v is the wave speed on the string. Now $v = (\tau/\mu)^{1/2}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f = (n/2L)(\tau/\mu)^{1/2}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)(\tau/\mu)^{1/2}$ and

$$f_2 = \frac{n_1 + 1}{2L} \sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L} \sqrt{\frac{\tau}{\mu}} + \frac{1}{2L} \sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L} \sqrt{\frac{\tau}{\mu}}.$$

This means $f_2 - f_1 = (1/2L)(\tau/\mu)^{1/2}$ and

$$\tau = 4L^2\mu(f_2 - f_1)^2$$

$$= 4(0.300)^2(0.650 \times 10^{-3})(1320 - 880)^2 = 45.3 \text{ (N)}.$$

45. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass-spring oscillator is proportional to $1/m^{1/2}$). Since the beat frequency also decreases the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

47. Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is $f = (1/2L)(\tau/\mu)^{1/2}$, where τ is the tension in the wire and μ is the linear mass density of the wire. Suppose the tension in one wire is τ and the oscillation frequency of that wire is f_1 . The tension in the other wire is $\tau + \Delta\tau$ and its frequency is f_2 . You want to calculate $\Delta\tau/\tau$ for $f_1 = 600$ Hz and $f_2 = 606$ Hz. Now, $f_1 = (1/2L)(\tau/\mu)^{1/2}$ and $f_2 = (1/2L)[(\tau + \Delta\tau)/\mu]^{1/2}$, so $f_2/f_1 = [(\tau + \Delta\tau)/\tau]^{1/2} = [1 + (\Delta\tau/\tau)]^{1/2}$. This leads to $\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [606/600]^2 - 1 = 0.020$.

51. Using the notation rule for choosing \pm signs in the Doppler effect formula, Eq. 17-47, discussed in §17-10, we have $v = 343$ m/s, $v_D = 2.44$ m/s, $f' = 1590$ Hz and $f = 1600$ Hz. Thus,

$$f' = f \frac{v + v_D}{v + v_S} \Rightarrow v_S = \frac{f}{f'}(v + v_D) - v = 4.61 \text{ m/s}.$$

54. We denote the speed of the French submarine by u_1 and that of the U.S. sub by u_2 . (a) The frequency as detected by the U.S. sub is

$$f_1' = f_1 \frac{v + u_2}{v - u_1} = (1000 \text{ Hz}) \frac{5470 + 70}{5470 - 50} = 1.02 \times 10^3 \text{ Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be $f_r = f_1(v + u_2)/(v - u_2)$. Since the French sub is moving towards the reflected signal with speed u_1 , then

$$f_1' = f_r \frac{v + u_1}{v} = f_1 \frac{(v + u_1)(v + u_2)}{v(v - u_2)}$$

$$= (1000 \text{ Hz}) \frac{(5470 + 50)(5470 + 70)}{5470(5470 - 70)} = 1.04 \times 10^3 \text{ Hz}.$$

57. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f' \frac{v + u_{\text{bat}}}{v - u_{\text{bat}}} = (3.9 \times 10^3 \text{ Hz}) \frac{v + v/40}{v - v/40}$$

$$= 4.1 \times 10^4 \text{ Hz}.$$

77.* The siren is between you and the cliff, moving away from you and towards the cliff. Both “detectors” (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17-47 (and see the discussion in the text- book

immediately after that equation regarding the selection of \pm signs). The source is the siren with $v_S = 10$ m/s. The problem asks us to use $v = 330$ m/s for the speed of sound. (a) With $f = 1000$ Hz, the frequency f_y you hear becomes

$$f_y = \frac{v + 0}{v + v_S} f = 970.6 \text{ Hz} \Rightarrow 9.7 \times 10^2 \text{ Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = \frac{v + 0}{v - v_S} f = 1031.3 \text{ Hz} \Rightarrow 1.0 \times 10^3 \text{ Hz}.$$

(c) The beat frequency is $f_c - f_y = 60$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

Ex.1-2: Pb.17-76.

Pb.21-1, 5, 6, 7, 10, 20, 26, 52, 66 (tentatively)

music sound 樂音; quality, timbre 音質(音色、音品); pitch, tone 音調(音高); octave 八度音; equal-tempered/diatonic scale 等程/自然音階; Middle C 中央 C; flute 長笛, vertical bamboo flute 簫, pipe panpipe 排笛, oboe 雙簧管, pipe organ 管風琴, horn; French horn 法國號, Bass/Baritone/Tenor/Alto/Soprano saxophone 低/上低/次中/中/高音薩克斯管, fujara 福加拉管; marimba 木琴(馬林巴); xylophone 木琴; noise 噪音; Vienna 維也納; Doppler 都普勒; Metallica 金屬製品合唱團;

sound wave 聲波; infrasonic/audible/ultrasonic sound 亞, 次聲/可聞聲/超音波; planar/spherical wave 平面/球面波; wave front 波前; pressure amplitude 壓力振幅; intensity 強度; sound level 音量級; Decibel scale 分貝標; loudness 響度; beat 拍; siren 警笛; path length difference 路程差; shock wave 震波; Mach number/cone (angle) 馬赫數/錐(角), subsonic 次(亞)音速; supersonic 超音速; hypersonic 超高音速; sonic boom 音暴; closed tube 閉管; open tube 開管; closed end 開放端; open end 封閉端; sonar 聲納; syrinx 耳咽管; Emperor penguin 國王企鵝;

S1. <生物可辨識聲音最大頻率 kHz >

狗: 30; 青蛙: 50; 蟋蟀: 100; 蝙蝠: 120。

S2. 人耳可藉聲音進入左右兩耳之時間差線索以辨識聲音的方向(cf. S.P.17-1)

S3. 理想氣體 $p = nRT/V = \rho(RT/M)$ 又 $B = \gamma p$ (單原子 $\gamma = 5/3$, 雙原子 $\gamma = 7/5$)

S4. $\Delta p_m = 30 \text{ Pa}$, $I = (1/2)(30 \text{ Pa})^2 / [(1.22 \text{ kg/m}^3)(346 \text{ m/s})] = 1.1 \text{ (W/m}^2)$; $\Delta p_m = 30 \times 10^{-6} \text{ Pa}$, $I = 1.1 \times 10^{-12} \text{ W/m}^2$;

單位 W/m^2 有時太大, 改用混合單位 W/cm^2 較方便。
 Ref. 蔡振家, 泛音唱法的物理基礎管樂器發音原理, 科學月刊 32#3, p.209, 民 90, 3 月。

重點整理—第 17 章 波動—II

聲波(聲音)：泛指縱波，或狹義的指人耳可鑑識的波段之縱波。①可聞聲： $f: 20 \text{ Hz} \sim 20 \text{ kHz}$ (Ear: 對 3 kHz 最靈敏)；②亞、次聲： $f < 20 \text{ Hz}$ ；③超音波： $f > 20 \text{ kHz}$ 。

波速(縱波) $v = \sqrt{B/\rho}$ ， $\rho =$ 介質之密度、 $B =$ 體彈性係數。例如水： $\rho = 0.998 \text{ g/cm}^3 (20^\circ\text{C})$ 、 $B = 2.2 \times 10^9 \text{ Pa} \Rightarrow v = 1485 \text{ m/s}$ ； $v(\text{air}) = 343 \text{ m/s}$ 。

聲速(氣體) $v = \sqrt{\gamma p/\rho} = \sqrt{\gamma RT/M}$ ， $R \equiv$ 氣體常數 ($8.314 \text{ J/mol}\cdot\text{K}$)、 $M \equiv$ 分子量、 $\gamma \equiv C_p/C_v$ 、 $T =$ 絕對溫度(K)， $p =$ 壓力。理想氣體 $B = \gamma p$ (單原子 $\gamma = 5/3$ ，雙原子 $\gamma = 7/5$)，^{Note} 聲波傳播為一絕熱過程， $B = \gamma p$ 、 $\gamma \equiv C_p/C_v$ 、 $C_p(C_v) =$ 定壓(定容)之比熱；若等溫過程， $B = p$ 。◆**聲速(空氣)** $v = 331.45 + 0.61 T_c (\text{m/s})$

縱波之波函數—表示縱向位移

$$s(x, t) = s_m \cos(kx - \omega t), s_m = \text{位移振幅}$$

常用**壓力改變量**以表示 $\Delta p(x, t) = \Delta p_m \sin(kx - \omega t)$ ， $\Delta p = -B(dy/dx) = kB s_m \sin(kx - \omega t)$ ，**壓力振幅**(最大壓力改變量) $\Delta p_m = kB s_m = (v\rho\omega) s_m (\ll p_0)$ ，與位移之相位差為 $\pi/2$ 。

波強度：通過垂直於波傳播方向之截面上單位面積的平均功率，簡言之，**波強度為每單位面積傳輸平均功率**。(J/s·m² or W/m²，混合單位 W/cm² 在音響學家間廣泛地使用)

對於圓柱管內行進(諧和)聲波，其縱向位移

$$s(x, t) = s_m \cos(kx - \omega t), \Delta p = (\rho v \omega) s_m \sin(kx - \omega t), Fv/A = pv = (p_0 + \Delta p)(\Delta s/\Delta t), \text{Note } (\omega/k)^2 = v^2 = B/\rho.$$

強度 $I = \langle vp \rangle_t = \langle v \Delta p \rangle = \rho v \omega^2 s_m^2 \langle \sin^2(kx - \omega t) \rangle$

$$I = \frac{1}{2} \rho v \omega^2 s_m^2 = \frac{1}{2} (\rho B)^{1/2} \omega^2 s_m^2.$$

強度通常以壓力振幅(易量測)表示較方便

$$I = \omega \Delta p_m^2 / 2kB = v \Delta p_m^2 / 2B = \Delta p_m^2 / 2\rho v = \Delta p_m^2 / (2\rho B)^{1/2}.$$

離點波源 r 處之**波強度**(當點波源之功率 P_s)

$$I = P_s / 4\pi r^2 \text{ (平方反比)}$$

◆人耳能忍受最大振幅的聲波之強度(壓力振幅為 30 Pa)為 $I = 1.1 \text{ W/m}^2$ (J/s·m²)。人耳能聽到最微弱聲音之**壓力振幅**為 $3 \times 10^{-5} \text{ Pa}$ ，對應的強度為 $\sim 10^{-12} \text{ W/m}^2$ 。

音量級(聲波) $\beta = (10 \text{ dB}) \log_{10}(I/I_0)$ ，(單位 dB) ◆**參考強度** $I_0 = 10^{-12} \text{ W/m}^2$ 。(聽到聲音大小並非正比於強度，而約略正比於強度的對數值，再者人耳可聽到的聲音強度範圍極大 $10^{-12} \text{ W/m}^2 \sim 1 \text{ W/m}^2$)

此資料專為教學用請勿流傳—楊志信
聽力損傷為①永久性的傷害，終生無法修復；②漸進式的，平時不易察覺！(噪音逾 70 dB ，對人體有害)

$$s_m \Rightarrow \Delta p_m \Rightarrow I \Rightarrow \beta$$

圓柱管內**駐波圖案** **開放端**：為壓力波節或位移波腹；**封閉端**：為位移波節或壓力波腹。**開管**：兩端皆開放端，**閉管**：一端開放而另一端封閉。**開管**之共振頻率 $f = v/\lambda = n(v/2L)$ ， $n = 1, 2, 3, 4, \dots$
閉管之共振頻率 $f = v/\lambda = n(v/4L)$ ， $n = 1, 3, 5, \dots$ 。
①管短 \Rightarrow 頻率高；②開管之共振頻率類似於兩端固定之弦；③閉管基頻較低，為開管的一半；④閉管音色較單調，因泛音較少！

管樂器基本發音原理：利用栓塞或壓住“音孔”以改變氣柱長度，按住“音孔”時氣柱較長，為低音；反之，則高音！^{Note} Middle C: 264 Hz , A: 440 Hz ;

拍 當兩聲音波干涉時，如頻率接近，合成波之振幅會呈週期性變化，此振幅變化引起響度變化，稱之。 $s_1 = s_m \cos(\omega_1 t)$ & $s_2 = s_m \cos(\omega_2 t)$ ， $s(t) = s_1 + s_2 = s_m \cos(\omega_1 t) + s_m \cos(\omega_2 t) = 2s_m \cos(\alpha t) \cos(\beta t)$ ，where $\alpha = (1/2)(\omega_1 - \omega_2)$ ， $\beta = (1/2)(\omega_1 + \omega_2)$ 。

拍頻 $f_{\text{beat}} = |f_1 - f_2|$ ^{Note} a) 人耳能鑑別之拍頻約 $6 \sim 7 \text{ Hz}$ ，b) 樂器調音時可利用拍現象。

都普勒效應 指當波源與受信者之間有相對運動時，所造成的波頻率變化(靠近變大、遠離變小)，(v : 聲速 v_S : 波源速率， v_D : 偵測器速率)

$$f' = f(v \pm v_D) / (v \pm v_S), v, v_S, v_D > 0.$$

馬赫數：飛行物之速率與空氣中聲速的比值。

①次(亞)音速：1 馬赫以下，客機 $\sim \text{Mach } 0.85$ ；②超音速：1~5 馬赫；③超音速：5 馬赫以上。

^{Note} IDF: 1.8, F16A/B: 2, F16: >2, 幻象: 2.2, Su-30: 2.4, SR-71: 3.2 -- 3500 km/h, 協和客機: 2.03。(X-43A: 7—7,700 km/h, 2004/03/27)

樂音三要素：1). **音調**(音高)—聲音的高低；2). **音質**(音色、音品)：諧音數目及其相對振幅(或強度)構成。樂器彈奏時，並非單頻聲音；高手與新手彈奏所發出聲音不同，即音色不同；而不同類樂器發出不同音色之樂音；3). **響度**—聲音大小。另有 4). 音的長短—音符及 5). 音的強弱—拍子。**樂音**(指聲樂及器樂)：由規律性重覆振動所產生的聲音；反之稱為**噪音**。

那麼企鵝於數千隻成群中如何找出其配偶？

例. $v = [(1.4 \times 8.314 \times 300) / 28.8 \times 10^{-3}]^{1/2} = 348 \text{ (m/s)}$,

$T = 300 \text{ K}$; $v = 332 \text{ m/s}$ $T = 273 \text{ K}(0^\circ \text{ C})$, air #

例. $v = [(1.66 \times 8.314 \times 273) / 20.18 \times 10^{-3}]^{1/2} = 432 \text{ (m/s)}$,

Ne (單原子, 0° C) #

例. $I = 10^{-5} \text{ W/m}^2$, $\beta = 10 \log(10^{-5} / 10^{-12}) = 70 \text{ (dB)}$.

$\beta = 35.0 \text{ dB} \Rightarrow I = I_0 10^{\beta/10} = 10^{-12} \times 10^{3.5} = 3.16 \times 10^{-9} \text{ (W/m}^2)$ [$\beta = 10 \log(I/I_0)$, $10^{3.5} = 3162$].

例. 某長度 95.0 cm 之圓柱管子可發出兩連續頻率 501 Hz 及 668 Hz 之諧音, 試決定其基頻; 此管子為一端或兩端皆開口(啟)的? 當時的聲速為何? **Sol.** $f_1 = 668 - 501 = 167 \text{ (Hz)} = f_1$ or $2f_1$, $501/167 = 3$ & $668/167 = 4 \Rightarrow \text{Open Tube!}$ 又

$v = (2L)(f_1) = 317 \text{ m/s}$ #

例]. 「泛音唱法」為歌唱藝術之一, 其以口腔為共鳴腔, 若將口腔視為一長 17 cm 之圓柱管, 其一端為嘴巴, 而另一端為聲帶, 則理論上此口腔之發音頻率為何? 設聲速為 340 m/s 。

Sol. 視口腔為一閉管:

$L = 0.17 \text{ m}$, $f_1 = v/4L = 340 / (4 \times 0.17) = 500 \text{ (Hz)}$ 。

例]. 蝙蝠利用回聲定位法來定位及辨識方向以獵捕食物: 蝙蝠發出 60.0 kHz 之聲波, 經獵物反射後, 蝙蝠接收到 61.8 kHz , 試計算蝙蝠之飛行速率值? 假設獵物靜止, 而聲速 $v = 340 \text{ m/s}$ 。

Sol. $f'(v-v_b) = f(v+v_b)$ or $(f'-f)v = (f'+f)v_b$

$$\Rightarrow v_b = 5.02 \text{ m/s} \#$$

07.* If d is the distance from the location of the earthquake to the seismograph and v_s is the speed of the S waves then the time for these waves to reach the seismograph is $t_s = d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

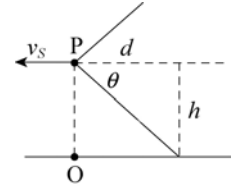
$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

so

$$d = \frac{v_s v_p \Delta t}{v_p - v_s} = \frac{(4.5)(8.0)(3.0 \times 60)}{8.0 - 4.5} = 1.93 \times 10^3 \text{ km}.$$

We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

63.* (a) The half angle θ of the Mach cone is given by $\sin \theta = v/v_s$, where v is the speed of sound and v_s is the speed of the plane. Since $v_s = 1.5v$, $\sin \theta = v/1.5v = 1/1.5$. This means $\theta = 42^\circ$. (b) Let h be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance d behind the plane. The situation is shown on the diagram below, with P indicating the plane and O indicating the observer. The cone angle is related to h and d by $\tan \theta = h/d$, so $d = h/\tan \theta$.



The shock wave reaches O in the time the plane takes to fly the distance d : $t = d/v = h/v \tan \theta = (5000 \text{ m}) / [1.5 \times (331 \text{ m/s}) \times \tan 42^\circ] = 11 \text{ s}$.

19.* Building on the theory developed in §17-5, we set $\Delta L/\lambda = n - 1/2$, $n = 1, 2, 3, \dots$, in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f_{\min, n} = \frac{(2n-1)v}{2\Delta L} = (n - \frac{1}{2})(286 \text{ Hz}),$$

where we have used $v = 343 \text{ m/s}$ (note the remarks made in the textbook at the beginning of the exercises and *Pbs* section) and $\Delta L = (19.5 - 18.3) \text{ m} = 1.2 \text{ m}$. (a) The lowest frequency that gives destructive interference is ($n = 1$) $f_{\min, 1} = (1 - 1/2)(286 \text{ Hz}) = 143 \text{ Hz}$. (b) The second lowest frequency that gives destructive interference is ($n = 2$) $f_{\min, 2} = (2 - 1/2)(286 \text{ Hz}) = 429 \text{ Hz} = 3f_{\min, 1}$. So the factor is 3. (c) The third lowest frequency that gives destructive interference is ($n = 3$) $f_{\min, 2} = (3 - 1/2)(286 \text{ Hz}) = 715 \text{ Hz} = 5f_{\min, 1}$. So the factor is 5. Now we set $\Delta L/\lambda = 1/2$ (even numbers) — which can be written more simply as “(all integers $n = 1, 2, \dots$)” — in order to establish constructive interference. Thus, $f_{\max, n} = n v / \Delta L = n(286 \text{ Hz})$. (d) The lowest frequency that gives constructive interference is ($n = 1$) $f_{\max, 1} = 286 \text{ Hz}$. (e) The second lowest frequency that gives constructive interference is ($n = 2$) $f_{\max, 2} = 2(286 \text{ Hz}) = 572 \text{ Hz} = 2f_{\max, 1}$. Thus, the factor is 2. (f) The third lowest frequency that gives constructive interference is ($n = 3$) $f_{\max, 3} = 3(286 \text{ Hz}) = 858 \text{ Hz} = 3f_{\max, 1}$. Thus, the factor is 3.

•備忘錄•