Chapter 21 ***Electric Charge***

01. (**a**) With a being the magnitude of acceleration, Newton's second and third laws lead to

$$m_2 a_2 = m_1 a_1 \Rightarrow$$

$$m_2 = \frac{(6.3\times10^{-7})(7.0)}{9.0} = 4.9\times10^{-7} \text{ (kg)}.$$

(**b**) The magnitude of the (only) force on particle 1 is

$$F_2 = m_1 a_1 = k\frac{|q_1 q_2|}{r^2} = (8.99\times10^9)\frac{|q|^2}{0.0032^2}.$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1\times10^{-11}$ C.

05. The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2},$$

where r is the distance between the charges. We want the value of q that maximizes the function $f(q) = q(Q-q)$. Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

06. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice). (**a**) The x-component of the force experienced by $q_1 = Q$ is

$$F_{1x} = \frac{1}{4\pi\varepsilon_0}\left[-\frac{(Q)(Q)}{2a^2}\cos45° + \frac{|q|Q}{a^2}\right]$$

$$= \frac{|q|Q}{4\pi\varepsilon_0 a^2}\left(1 - \frac{Q/|q|}{2\sqrt{2}}\right),$$

which (upon requiring $F_{1x} = 0$) leads to $Q/|q| = 2\sqrt{2}$, or $Q/q = -2\sqrt{2} = -2.83$.

(**b**) The y-component of the net force on $q_2 = q$ is

$$F_{2y} = \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q|^2}{2a^2}\sin45° - \frac{|q|Q}{a^2}\right]$$

$$= \frac{|q|^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2\sqrt{2}} - \frac{Q}{|q|}\right),$$

which (if we demand $F_{2y} = 0$) leads to $Q/q = -1/2\sqrt{2}$. The result is inconsistent with that obtained in part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces present are given by Eq. 21-1.

10. With rightwards positive, the net force on q_3 is

$$F_3 = F_{31} + F_{32} = \frac{kq_1q_3}{(L_{12}+L_{23})^2} + \frac{kq_2q_3}{L_{23}^2}.$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero $L_{23} = L_{12}$ and canceling k, q_3 and L_{12} leads to

$$\frac{q_1}{4.00} + q_2 = 0 \quad\Rightarrow\quad \frac{q_1}{q_2} = -4.00.$$

20. If θ is the angle between the force and the x-axis, then $\cos\theta = x/(x^2+d^2)^{1/2}$. We note that, due to the symmetry in the problem, there is no y component to the net force on the third particle. Thus, F represents the magnitude of force exerted by q_1 or q_2 on q_3. Let $e = +1.60\times10^{19}$ C, then $q_1 = q_2 = +2e$ and $q_3 = +4e$ and we have

$$F = \frac{1}{4\pi\varepsilon_0}\frac{2(2e)(4e)}{x^2+d^2}\frac{x}{\sqrt{x^2+d^2}} = \frac{1}{\pi\varepsilon_0}\frac{4e^2x}{(x^2+d^2)^{3/2}}.$$

(**a**) To find where the force is at an extremum, we can set the derivative of this expression equal to zero, $dF/dx = 0$, and solve for x, but it is good in any case to graph the function for a fuller understanding of its behavior – and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum [and will be presented in part (b)] and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x = 0$, which is the smallest value of the net force in the interval $5.0 \text{ m} \ge x \ge 0$. (**b**) The maximum is found to be at $x = d/2^{1/2}$ or roughly 12 cm. (**c**) The value of the net force at $x = 0$ is $F_{net} = 0$. (**d**) The value of the net force at $x = d/2^{1/2}$ is $F_{net} = 4.9\times10^{26}$ N.

52. (**a**) Since $q_A = -2.00$ nC and $q_C = +8.00$ nC, Eq. 21-4 leads to

$$|\vec{F}_{AC}| = \frac{|q_Aq_C|}{4\pi\varepsilon_0 d^2} = 3.60\times10^{-6} \text{ (N)}$$

$$= \frac{(8.99\times10^9)(2.00\times10^{-9})(8.00\times10^{-9})}{(0.200)^2}.$$

(**b**) After making contact with each other, both A and B have a charge of

$$\frac{q_A+q_B}{2} = \frac{-2.00-4.00}{2} = \_3.00 \text{ (nC)}.$$

When B is grounded its charge is zero. After making contact with C, which has a charge of +8.00 nC, B acquires a charge of [0+(–8.00 nC)]/2 = –4.00 nC, which charge C has as well. Finally, we have Q_A = –3.00 nC and $Q_B = Q_C$ = –4.00 nC. Therefore,

$$|\vec{F}_{AC}| = \frac{|q_Aq_C|}{4\pi\varepsilon_0 d^2} = 2.70\times10^{-6} \text{ (N)}$$

$$= \frac{(8.99\times10^9)(3.00\times10^{-9})(4.00\times10^{-9})}{(0.200)^2}$$

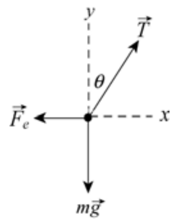
(**c**) We also obtain

$$|\vec{F}_{AC}| = \frac{|q_Aq_C|}{4\pi\varepsilon_0 d^2} = 3.60\times10^{-6} \text{ (N)}$$

$$= \frac{(8.99\times10^9)(4.00\times10^{-9})(4.00\times10^{-9})}{(0.200)^2}.$$

Thus, the magnitude is $|q| = 2.4\times10^{-8}$ C.

66.• (**a**) A force diagram for the left ball is shown below. The force of gravity $\boldsymbol{mg}$ acts downward, the electrical force $\boldsymbol{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T\cos\theta - mg = 0$ and the x component yields $T\sin\theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$. Examination of the geometry of Fig. 21-43 leads to



$$\tan\theta = (x/2)/\sqrt{L^2-(x/2)^2}\,.$$

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = q^2/(4\pi\varepsilon_0 x^2),$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \Rightarrow x \approx \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(**b**) With Eq. 21-5, we solve $x^3 = q^2L/(2\pi\varepsilon_0 mg)$ for the charge:

$$q = \left(\frac{mgx^3}{2k_eL}\right)^{1/2} = \sqrt{\frac{(0.010)(9.8)(0.050)^3}{2(8.99\times10^9)(1.20)}} = \pm2.4\times10^{-8}\,(\text{C}).$$

25.* The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37\times10^6\text{ m})^2 = 5.1\times10^{14}\text{ m}^2$ would be

$$i = (5.1\times10^{14}14\text{ m}^2)(1500\text{ protons/s}\cdot\text{m}^2)$$
$$(1.6\times10^{-19}\text{ C/proton}) = 0.12\text{ (A)}.$$

26.* The volume of 250 cm^3 corresponds to a mass of 250 g since the density of water is 1.0 g/cm^3. This mass corresponds to 250/18 = 14 moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O, so $Q = 14N_Aq = 14(6.02\times10^{23})(10)(1.60\times10^{-19}\text{ C}) = 1.30\times10^7$ C.

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

Ex.2-1: Pb.21-7. *Pb.**22**-1, 3, 11, 19, 22, 27, 29, 31, 38, 43, 55, 84, 86, 87 (tentatively).* •**備忘錄**•

現代電子儀器如何成為細菌污染源？

$k_e = 8.99\times10^9$ N·m^2/C^2, $e = 1.60\times10^{-19}$ C.

重點整理－第 21 章 電 荷

電荷：物質之性質，它能引起物質產生或感受到電與磁之效應。♦**庫侖定律**：兩點電荷 q_1 & q_2 間的作用力(靜電力)與兩點電荷乘積 q_1q_2 成正比，與兩點電荷間距離 r_{12} 平方成反比，作用力方向為沿兩點電荷之連線方向，(法人庫侖於 1785 年提出)

$$\vec F_{21} = -\vec F_{12} = k_e q_1q_2\,\hat r_{12}/r_{12}^2。$$

$k_e = 1/4\pi\varepsilon_0$ (靜電常數)，$\varepsilon_0 = 8.854187817\times10^{-12}$ C^2/N·m^2 (真空之電容率)，適用於①點電荷，②靜止的電荷。***Note*** 電荷有兩類，兩者同號相斥，異號則相吸但質量只有一種，兩者間皆相吸。

庫倫與安培：電荷之單位為 C (庫倫),電流之單位為 A (安培), 1 C = (1 A)(1 s), 1 C 為於載電流 1 A 之導線內在 1 s 內通過某截面之電量，安培為 SI 之基本單位，此導源於電流較電荷更易精準量測。

庫侖力之**疊加原理**： $\boldsymbol{F}_1 = \boldsymbol{F}_{12} + \boldsymbol{F}_{13} + \ldots + \boldsymbol{F}_{1N}$。

殼層定理：***A***).對一電荷均勻分佈之球殼而言，其與殼內電荷之靜電力作用為零。***B***).對一電荷均勻分佈之球(球殼)而言，其與球(球殼)外電荷之靜電力作用儼然整個球(球殼)之電荷均集中於球(殼)心。***C***).對一電荷均勻分佈之球而言，其與球內點電荷之電力作用與點電荷到球心距離成正比。

Note 金屬球帶電時如同帶電薄球殼。

原子由**原子核**(帶正電)及核外**電子**(帶負電)組成，**原子核**由**質子**(帶正電)及**中子**(電中性)組成

原子分子 ⇒ 物質物體 ⇒ 大觀世界 ⇒ 宇宙

導體：物質內具有大量可自由運動的帶電質點者(金屬及特殊材料等)；**絕緣體**：物質內帶電質點無法自由運動者(塑膠、橡膠、玻璃及陶瓷等)；**半導體**：導電性介於導體與絕緣體之間者(矽鍺等)，其導電性可人為地特意調配；**超導體**：零電阻之物質(Y-Ba-Cu, 98 K; La-Ba-Cu, 35 K; NbGe, 23 K; α-Hg, 4.15 K)。 $e\,(>0) = 1.602176462\times10^{-19}$ C.

電荷量子化：$\Delta q = ne$，自然界所有物體之帶電量為 e 或 e 之整數倍; **基本電量**(電子帶電量) e.

電荷守恒：在任何物理過程，沒有正電荷或負電荷被產生或消滅；**宇宙間電荷之代數和為定值**；配對產生 $\gamma \to e^- + e^+$；消滅過程 $e^+ + e^- \to \gamma + \gamma$。核分裂反應 $U^{238} \to Th^{234} + He^4$；•**備忘錄**•

電磁學發展史

*600 BC 左右，米利都之台利斯注意到**琥珀**和**羊皮**互相摩擦後，琥珀能吸引羽毛或稻草等輕物體。亦報告 800 BC 前，磁鐵相吸之事。

amber (琥珀) → elektron (希文)

*西元前 1st 世紀，詩人 Lucretius (小亞細亞之美格尼西亞)，發現指路石。

*4 世紀，意大利水手注意到 St. Elmo's fire (桅頂電光，尖端放電現象)。

*11 世紀，中國人及阿拉伯水手 — 利用指南針

大航海時代(15th,1492AD 哥倫布發現新大陸)。

*17 世紀(1600 AD)，吉爾伯特(英人)首先區別"電"與"磁"現象。

* 18 世紀，電現象—帶電物體(含人體)之實驗研究盛行。

*1733, 杜菲(法人)猜測兩種'電荷'存在—電流體

*1746, 萊登瓶(充電瓶 or 電容器)，荷蘭萊登大學穆申部魯克發明，為一裝水的玻璃瓶。

*1750, 富蘭克林(美人)提議命名"**正**"及"**負**"電，得到電流體的物體帶正的。

*1785, **庫倫定律**，庫侖(法人, 1736–1806)；電荷 q 單位 C。

*1800, 伏打電堆，伏打(義大利人)；電位差或電壓單位: 伏特 V。

***高斯定律**，高斯(德人,1777~1855, 14~17 歲)；磁場 B 單位: G。

*1820, 電生磁(**電流磁效應**)，厄斯特(丹麥)，通電流之導線使其旁之指南針偏向；磁場 H 單位: Oe。

*1820~7, 電動力學現象的數學基礎，安培(法人)，安培(右手)定則、**安培定律**；電流 i 單位: A。

*1827, 歐姆定律，歐姆(德人,1789-1854)，電阻 R 單位Ω。

*1831, **電磁感應定律**(磁生電)，法拉第(英人)，"場"及"力線"等觀念，第一台發電機；電容 C 單位:F。

*1833, 楞次定律—**電磁感應定律**(定性), 楞次(俄人)。

*亨利(美人)，磁生電—**電磁感應**；電感 L 單位 H

*1855~1868 (Mid-19th),電磁理論，馬克士威(英人)，統合電學與磁學理論為四個方程式，預測電磁波存在；磁通量 Φ 單位(cgs):Mx。

*1888, 證實電磁波，赫芝(德人)，頻率 f 單位 Hz

*1897, 帶負電之電子，湯姆笙(英人)發現。

*1911, α粒子散射實驗，拉塞福(紐/英人,1871-1937)，帶正電原子核存在。

*1911, 超導體 Hg (α)發現，翁納斯(荷蘭人), (1986 高溫超導發現)。

*1909~1917, 電子之帶電量，密利根(美人)油滴實驗測得：物質帶電量子化。

electromagnetism 電磁學; electric charge 電荷; elementary charge 基本電荷; positive/negative charge 正/負電荷; quantized 量子化; conservation of charge 電荷守恆; Coulomb's law 庫侖定律; permittivity constant 電容率常數; electric force 電力; electric current 電流; atom 原子; proton 質子; neutron 中子; electron 電子; quark 夸克, pair production/annihilation 配對產生/消滅, conduction/free electron 傳導/自由電子; conductor 導體; ground 接地; semiconductor 半導體; insulator 絕緣體; superconductor 超導體; lodestone 指路石; aurora 極光; amber 琥珀; straw 稻草,麥稈; fur 毛皮, silk 絲巾, endoscopic 內視鏡的,surgery 外科手術; Ampère 安培, Coulomb 庫侖, Franklin 富蘭克林, Faraday 法拉第, Oersted 厄斯特, Maxwell 馬克士威,

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