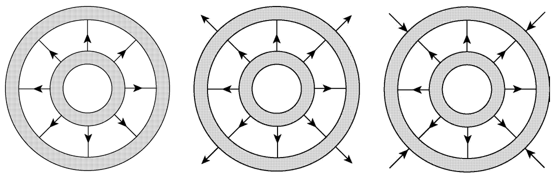
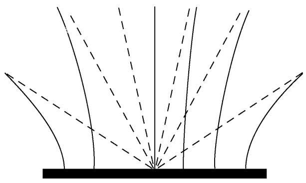
Chapter 22 **Electric Field**

01. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge, which resides on the larger shell. The following sketches are is for the cases $q_1 = q_2$ (left figure), $q_1 > q_2$ (middle figure), and $q_1 < q_2$ (right figure).



03. The following diagram is an edge view of the disk and shows the field lines above it. Near the disk, the lines are perpendicular to the surface and since the disk is uniformly charged, the lines are uniformly distributed over the surface. Far away from the disk, the lines are like those of a single point charge (the charge on the disk). Extended back to the disk (along the dotted lines of the diagram) they intersect at the center of the disk. If the disk is positively charged, the lines are directed outward from the disk. If the disk is negatively charged, they are directed inward toward the disk. A similar set of lines is associated with the region below the disk.



11. The x component of the electric field at the center of the square is given by, putting $d = a/2^{1/2}$,

$$E_x = \frac{\cos 45°}{4\pi\varepsilon_0}\left(\frac{|q_1|}{d^2}+\frac{|q_2|}{d^2}-\frac{|q_3|}{d^2}-\frac{|q_4|}{d^2}\right)$$

$$=\frac{\sqrt{2}}{4\pi\varepsilon_0 a^2}(|q_1|+|q_2|-|q_3|-|q_4|) = 0 .$$

Similarly, the y component of the electric field is

$$E_y = \frac{\cos 45°}{4\pi\varepsilon_0}\left(\frac{|q_2|}{d^2}+\frac{|q_3|}{d^2}-\frac{|q_1|}{d^2}-\frac{|q_4|}{d^2}\right)$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}(-|q_1|+|q_2|+|q_3|-|q_4|)$$

$$=\frac{\sqrt{2}(8.99\times10^9)(2.0\times10^{-8})}{0.050^2} = 1.02\times10^5 \text{ (N/C)}.$$

Thus, the electric field at the center of the square is

$$\vec{E} = E_y\,\mathbf{j} = (1.02\times10^5 \text{ N/C})\,\mathbf{j}.$$

22. We use Eq. 22-3, assuming both charges are positive. At P, we have

$$E_{\text{left}} = E_{\text{right}} \Rightarrow \frac{q_1 R}{(R^2+R^2)^{3/2}} = \frac{q_2(2R)}{[(2R)^2+R^2]^{3/2}} .$$

Simplifying, we obtain

$$q_1 / q_2 = (2)(2/5)^{3/2} = 0.506 .$$

24. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda\pi R$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E} = 2(-\hat{\mathrm{j}})\frac{\lambda\sin\theta}{4\pi\varepsilon_0 R}\Big|_{-90}^{90} = -\frac{q}{\pi^2\varepsilon_0 R^2}\hat{\mathrm{j}} .$$

(**a**) With $R = 8.50\times10^{-2}$ m and $q = 1.50\times10^{-8}$ C, $E_{\text{net}} = 23.8$ N/C. (**b**) The net electric field $\boldsymbol{E}_{\text{net}}$ points in the $-\mathbf{j}$ direction, or $-90°$ counterclockwise from the $+x$ axis.

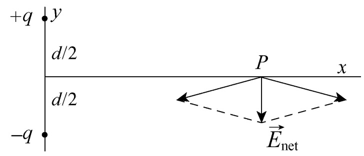
19. Consider the figure below. (**a**) The magnitude of the net electric field at point P is

$$E_{\text{net}} = 2E_1\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{qd}{[r^2+(d/2)^2]^{3/2}}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2+(d/2)^2}\frac{d/2}{[r^2+(d/2)^2]^{1/2}} .$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$E_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3} .$$



(**b**) From the figure, it is clear that the net electric field at point P points in the $-\mathbf{j}$ direction, or $-90°$ from the $+x$ axis. <*cf. Eq. 22-9*>

28. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. *S.P.* 22-4 illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle 2θ) is

$$E = \frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta - \sin(-\theta)] = \frac{\lambda\sin\theta}{2\pi\varepsilon_0 r} .$$

Now, the arc length is $L = 2r\theta$ if θ is expressed in radians. Thus, using R instead of r, we obtain

$$E = \frac{(Q/L)\sin\theta}{2\pi\varepsilon_0 R} = \frac{Q\sin\theta}{4\pi\varepsilon_0\theta R^2} .$$

Thus, with $\theta = \pi/2$, the problem asks for the ratio $E_{\text{ptl}}/E_{\text{arc}}$, where E_{ptl} is given by Eq.22-3. We obtain

$$\frac{Q/4\pi\varepsilon_0 R^2}{Q\sin\theta/2\pi^2\varepsilon_0 R^2} = \frac{\pi}{2} \approx 1.57 .$$

38. (**a**) $F_e = eE = (1.6\times10^{-19})(3.0\times10^6) = 4.8\times10^{-13}$ (N). (**b**) $F_{\text{ion}} = Eq_{\text{ion}} = E_e = 4.8\times10^{-13}$ N.

31. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right),$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1-\frac{z}{\sqrt{z^2+R^2}}=\frac{1}{2} \;\Rightarrow\; \frac{z}{\sqrt{z^2+R^2}}=\frac{1}{2}.$$

Squaring both sides, then multiplying them by z^2+R^2, we obtain $z^2 = z^2/4+R^2/4$. Thus, $z^2 = R^2/3$, or $z = R/3^{1/2}$. With R = 0.600 m, we have z = 0.346 m.

43. (**a**) We use $\Delta x = v_{av}t = (½)vt$:

$$v=\frac{2\Delta x}{t}=\frac{2(2.0\times10^{-2})}{1.5\times10^{-8}}=2.7\times10^{6}\ \text{(m/s)}.$$

(**b**) We use $\Delta x = (½)at^2$ and $E = F/e = ma/e$:

$$E=\frac{ma}{e}=\frac{2m\Delta x}{et^2}=1.0\times10^{3}\ \text{(N/C)}$$

$$=\frac{2(9.11\times10^{-31})(2.0\times10^{-2})}{(1.60\times10^{-19})(1.5\times10^{-8})^2}.$$

51. (**a**) The magnitude of the dipole moment is $p = qd$ = (1.50×10^{-9} C)(6.20×10^{-6} m) = 9.30×10^{-15} C·m. (**b**) Following the solution to part (c) of *S.P.* 22-6, we find $U = U(180°) - U(0) = 2pE$ = 2(9.30×10^{-15})(1100) = 2.05×10^{-11} (J).

70. The two closest charges produce fields at the midpoint which cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}\,d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result: (*cf. Pb.22-12*)

$$E=\frac{Q}{4\pi\varepsilon_0 r^2}=\frac{Q}{3\pi\varepsilon_0 d^2}.$$

90. Since both charges are positive (and aligned along the z axis) we have

$$E=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z+d/2)^2}+\frac{q}{(z-d/2)^2}\right].$$

For $z \gg d$, we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$E=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z^2}+\frac{q}{z^2}\right)=\frac{2q}{4\pi\varepsilon_0 z^2}.$$

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

Ex.2-2: Pb.22-25. *Pb.* **23**-*1, 7, 9, 16, 32, 34, 35, 39, 43, 49, 70, 72, 85* (*tentatively*).

field 場; electric field 電場; electric field lines 電場線; lines of electric force 電力線; electric dipole 電偶極(子);dipole axis 偶極軸; electric dipole moment/quadrupole 電偶極矩/電四極; uniform field 均勻場; action at a distance 超距作用, charged ring / disk 帶電環/圓盤, electric breakdown 電崩潰, ink-jet 噴墨, deflecting plate 偏向板; microwave oven 微波爐; sprite 紅色精靈; nagging 擾人的 *噴墨印表機，菲謝蒂，科學人 2003 年 2 月號。*噴墨列印--**科技與藝術的完美結合**，張棋榕，科學發展 366 期 (9206) 60。**Q&A.*使用微波爐真的對身體有害嗎？，科學人 2004 年 8 月號。*科學新發現--閃電大追蹤(Naked science: Lightning)，NGC。

簡言之，何者造成紅色精靈？

重點整理－第 22 章 電 場

場：表示某物理量於空間的分佈。

電場的定義：單位正電荷所受之電力，試驗電荷 q_0 (→ 0)受一電力 $\vec F$ 作用，則 q_0 處之電場 $\vec E = \vec F / q_0$，電場**單位**為 N/C or V/m。*Note* ***a***.電場與 q_0 存在有否無關，***b***.電場遵循疊加原理。

均勻電場：於一定區域內電場的大小及方向皆相等，即 $\vec E$ = const. ♦ **電場線**：將電場具(實)體化，利用圖形表示空間該點電場相對大小與方向。

電場線性質：***a***.靜電場線從正電荷出發，而結束於負電荷；***b***.從電荷發出或結束於電荷之場線數正比於電荷大小；***c***.場線的切線方向表示空間該點電場的方向；***d***.電場強度正比於場線密度(通過單位截面積之場線數目)；***e***.兩場線決不相交。*Note* 場線並非帶電質點之運動路徑。

點電荷 *q* 產生的電場：

$\vec E = k_e q\,\hat r / r^2$，$\vec r$ 為相對於 q 之位置向量。

電偶極：兩等量 q 但符號相反並相隔一距離 d 之電荷分佈，電偶極矩(物理量) $\vec p$ 為其性質，$p = qd$, $\vec p = q\vec d$，電偶極於遠處產生的電場 $E = f\,k_e p/r^3$，r ($r \gg d$)為觀測點至電偶極中心之距，f = 1 (偶極軸垂直平分線) or 2 (偶極軸)。

連續電荷分佈產生的電場：先考慮電荷元：一小段 $d\ell$ (線密度λ)或小面積 dA (面密度σ)或小體積 dV (體密度ρ)，視此電荷元為點電荷 dq，(= $\lambda d\ell$ or σdA or ρdV)，再將各電荷元對電場之貢獻疊加而得。***A***.圓弧(半徑 R 張角 2θ)電於弧心之電場

於對稱軸弧心上，$E=\dfrac{Q\sin\theta}{4\pi\varepsilon_0\theta R^2}$。

B.帶電薄圓環(半徑 R)產生的電場

於對稱軸(z)上，$E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{qz}{(z^2+R^2)^{3/2}}$。

C.帶電薄圓盤(半徑 R)產生的電場

於對稱軸(z)上，$E=\dfrac{\sigma}{2\varepsilon_0}\left[1-\dfrac{z}{(z^2+R^2)^{1/2}}\right]$。

電場中之點電荷：點電荷 q 所受之電力 $\vec F = q\vec E$，如電場為均勻的，則其作**等加速度運動**

電場中之電偶極：電偶極 p 所受之 ***a***.電力 $\vec F = 0$，***b***.力矩 $\vec\tau = \vec p \times \vec E$, ***c***)電位能 $U = -\vec p\cdot\vec E = -pE\cos\theta$，$\theta$為 $\vec p$ 與 $\vec E$ 之夾角。

86. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{(1.60\times10^{-19})(2.00\times10^{3})}{9.11\times10^{-31}} = 3.51\times10^{14}\ (\text{m/s}^2).$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_o t\cos\theta,\ y = v_o t\sin\theta - (½)at^2,$$

$$\text{and } v_y = v_o\sin\theta - at.$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0\sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} = 2.56\times10^{-2}\ (\text{m})$$

$$= \frac{(6.00\times10^6)^2\sin^2 45°}{2(3.51\times10^{14})}.$$

Since this is greater than d = 2.00 cm, the electron might hit the upper plate. (**b**) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0\sin\theta = (6.00\times10^6\ \text{m/s})\sin45° = 4.24\times10^6\ \text{m/s}$$

and

$$2ad = 2(3.51\times10^{14}\ \text{m/s}^2)(0.0200\ \text{m}) = 1.40\times10^{13}\ \text{m}^2/\text{s}^2.$$

the solution to $d = v_0 t\sin\theta - (½)at^2$ is

$$t = \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} = 6.43\times10^{-9}\ (\text{s})$$

$$= \frac{4.24\times10^6 - \sqrt{(4.24\times10^6)^2 - 1.40\times10^{13}}}{3.51\times10^{14}}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$x = v_o t\cos\theta = (6.00\times10^6\ \text{m/s})(6.43\times10^{-9}\ \text{s})\cos45° = 2.72\times10^{-2}\ \text{m}.$$

This is less than L so the electron hits the upper plate at x = 2.72 cm. •**備忘錄**•

58.* (**a**) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center. (**b**) The result (E = 0) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z^2$ as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r^2$ at distant points). (**c**) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{dE}{dz} = \frac{q(R^2 - 2z^2)}{4\pi\varepsilon_0(R^2+z^2)^{5/2}} = 0 \ \Rightarrow\ z = \frac{R}{\sqrt{2}}.$$

(**d**) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{\max} = 3.46\times10^7$ N/C.

電崩潰及**火花**　從太空或天然放射性來的宇宙射線將少數空氣分子游離，而這些游離分子若受到夠強的電場(> 3×10^6 N/C)加速而獲致足夠能量，將使電中性空氣分子游離，產生更多的空氣離子，因此空氣失去絕緣性質成為導體，而受激發的離子再與電子結合並釋放能量而發射光。

紅色精靈　是一種伴隨雷雨產生的高空大氣放電現象，閃光由下向上竄升。一般所看到的(白色)閃電是雷雨系統從雲層頂端往地面向下放電，其實雲層也會發生向上閃電，只是很難觀測到，巨大噴流就是其中之一。　「巨大噴流」，從距離海平面十五公里的雲層頂端，一直延伸到九十公里高的電離層，其為「紅色精靈」與「藍色噴流」的混合體。「紅色精靈」出現範圍從地表三十公里到九十公里高，寬度約五至十公里，可持續數十毫秒，「藍色噴流」是從雲層頂端到四十公里高。　產生紅色和藍色的原因，是大氣中氮分子和氮離子受幾十萬到上百萬伏特高壓電流激發所致。「巨大噴流」出現的同時，也會發出極低頻的無線電波，並使發生地點的「大域電流迴路」產生短路，降低電離層和地面之間的電壓差。此現象直到 1994 年因美國的研究，而被科學界確認存在。全世界有三個發生「紅色精靈」的地區，分別於亞洲大陸、南美亞馬遜河附近、非洲。我國華衛二號搭載「高空大氣閃電影像儀」可作高空閃電觀測。