Chapter 24 *Electric Potential*

02. The magnitude is

$$\Delta U = e\Delta V = 1.2\times10^{9}\ \text{eV} = 1.2\ \text{GeV}.$$

05. (**a**) $E = F/e = (3.9\times10^{-15}\ \text{N})/(1.60\times10^{-19}\ \text{C}) = 2.4\times10^{4}\ \text{N/C}$. (**b**) $\Delta V = E\Delta s = (2.4\times10^{4}\ \text{N/C})(0.12\ \text{m}) = 2.9\times10^{3}\ \text{V}$.

11. (**a**) The charge on the sphere is $q = 4\pi\varepsilon_0 VR = (200\ \text{V})(0.15\ \text{m})/(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2) = 3.3\times10^{-9}\ \text{C}$. (**b**) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = q/(4\pi R^2) = 3.3\times10^{-9}\ \text{C} / [4\pi(0.15\ \text{m})^2] = 1.2\times10^{-8}\ \text{C/m}^2.$$

14. According to the problem statement, there is a point in between the two charges on the x axis where the net electric field is zero and the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

17. (**a**) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\varepsilon_0 R$. Thus $R = q/4\pi\varepsilon_0 V = (30\times10^{-12}\ \text{C})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)/500\ \text{V} = 5.4\times10^{-4}\ \text{m}$. (**b**) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus, $V' = q'/(4\pi\varepsilon_0 R') = (2q)/[4\pi\varepsilon_0(2^{1/3}R)] = 2^{2/3}V = 2^{2/3}(500\ \text{V}) \approx 790\ \text{V}$.

23. (**a**) All the charge is the same distance R from C, so the electric potential at C is (in SI units)

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{5Q_1}{R} = -2.30\ \text{V}.$$

where the zero was taken to be at infinity. (**b**) All the charge is the same distance from P. That distance is $r_p = (R^2+D^2)^{1/2}$ so the electric potential at P is (in SI units)

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_p} - \frac{6Q_1}{r_p}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{5Q_1}{\sqrt{R^2+D^2}} = -1.78\ \text{V}.$$

30. The magnitude of the electric field is given by $|E| = |-\Delta V/\Delta x| = 2(5.0\ \text{V})/(0.015\ \text{m}) = 6.7\times10^{2}\ \text{V/m}$. At any point in the region between the plates, $\boldsymbol{E}$ points away from the positively charged plate, directly towards the negatively charged one.

39. (**a**) We use Eq. 24-43 with $q_1 = q_2 = -e$ and $r = 2.00$ nm:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$= \frac{(8.99\times10^{9})(1.60\times10^{-19})^2}{2.00\times10^{-9}} = 1.15\times\times10^{-19}\ \text{(J)}.$$

(**b**) Since $U > 0$ and $U \propto 1/r$ the potential energy U decreases as r increases.

37. We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a} - \frac{1}{a} + \frac{1}{\sqrt{2}a} - \frac{1}{a} - \frac{1}{a} + \frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{2}} - 2\right).$$

Thus the amount of work required to set up the system is given by (in SI units)

$$W = \Delta U = U_f - U_i = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{2}} - 2\right)$$

$$= \frac{2(8.99\times10^{9})(2.30\times10^{-12})^2}{0.640}(-1.293) = -1.92\times\times10^{-13}\ \text{(J)}.$$

53. If the electric potential is zero at infinity, then the potential at the surface of the conducting sphere is given by $V_r = q/4\pi\varepsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\varepsilon_0 rV_r = \frac{(0.15)(1500)}{8.99\times10^{9}} = 2.5\times\times10^{-8}\ \text{(C)}.$$

56. (**a**) Since the two conductors are connected V_1 and V_2 must be equal to each other. Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$, or (**b**) $q_1/q = 1/3 = 0.333$, (**c**) and $q_2/q = 2/3 = 0.667$. (**d**) The ratio of surface charge densities is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2.00\ .$$

81.• (**a**) Clearly, the net potential

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{|x|} + \frac{2q}{|d-x|}\right)$$

is not zero for any finite value of x. (**b**) The electric field cancels at a point between the charges:

$$\frac{q}{x^2} = \frac{2q}{(d-x)^2},$$

which has the solution: $x = (\sqrt{2}-1)\,d = 0.41$ m.

106.• We imagine moving all the charges on the surface of the sphere to the center of the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$V_R = V_\infty + \int_\infty^R E\,dr = \frac{q}{4\pi\varepsilon_0}\int_\infty^R \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0 R} = 8.43\times10^{2}\ \text{(V)}.$$

107.• On the dipole axis $\theta = 0$ or π, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}.$$

97.• Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V_R = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37\times10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E_R = q/4\pi\varepsilon_0 R^2$, so

$$V_R = E_R R = (100\ \text{V/m})(6.37\times10^6\ \text{m}) = 6.4\times10^8\ \text{V}.$$

112.• (**a**) The potential would be

$$V_e = \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e\sigma_e}{4\pi\varepsilon_0} = -0.12\ (\text{V})$$

$$= 4\pi(6.37\times10^6)(1.0)(-1.6\times\times10^{-19})(8.99\times10^9).$$

(**b**) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\ \text{V}}{6.37\times10^6\ \text{m}} = -1.8\times\times10^{-8}\ \text{N/C},$$

or $|E| = 1.8\times10^{-8}$ N/C. (**c**) The minus sign in E indicates that $\boldsymbol{E}$ is radially inward.

114.• (**a**) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = (z^2+R^2)^{1/2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}}$$

$$= \frac{1}{4\pi\varepsilon_0\sqrt{z^2+R^2}}\int dq = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}.$$

(**b**) The electric field is along the axis and its component is given by

$$E = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)\frac{2z}{(z^2+R^2)^{3/2}} = \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

電偶極: $V = k_e p\dfrac{\cos\theta}{r^2}$, $k_e = \dfrac{1}{4\pi\varepsilon_0}$,

線段電荷(L,λ)端點外 y: $V = k_e\lambda\ell n\dfrac{L+\sqrt{L^2+y^2}}{y}$.

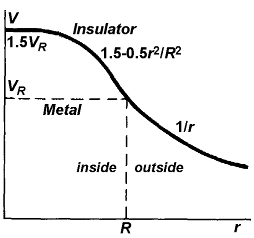
環(半徑 R)均勻帶電 Q: $V = k_e Q\dfrac{1}{(z^2+R^2)^{1/2}}$.

盤(半徑 R)均勻帶電σ: $V = 2\pi k_e\sigma(\sqrt{z^2+R^2} - |z|)$.

球(半徑 R)內均勻帶電 Q:

$V = k_e\dfrac{Q}{r}$ (球外),

$V = k_e\dfrac{Q}{2R}(3-\dfrac{r^2}{R^2})$(球內).



均勻電場($\boldsymbol{E}$ 朝$-x$): $V = E\,x$.

electric potential energy 電位能;
electric potential 電位; equipotential surface 等位面; volt 伏特; electroevolt (eV)電子伏特; polarize 極化; induced 感應的; spark/corona discharge 火花/電暈放電; nonspherical 非球形的;

S1. $\int(x^2+a^2)^{-1/2}dx = \ell n[x+(x^2+a^2)^{1/2}]$.

S2. $\int x(x^2+d^2)^{-1/2}dx = (x^2+d^2)^{1/2}$.

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重點整理－第 24 章 電 位

電位能 點電荷系統之**電位能**為將各電荷以等速率從相距無窮遠移至給定點所須之外部功。

電位 空間各點之**電位**為將單位(正)電荷以等速率從無窮遠處(零電位)移至給定點所須之外部功(單位 V = J/C)， $\Delta V = V - 0 = W_{ext} = -\int\boldsymbol{E}\cdot d\boldsymbol{s}$。

絕對電位:將零電位定於無窮遠處所得的電位。

Note **沿著場線前進，電位減小**。水往低處流。

電子伏特(eV)為能量單位，

$$W = q\Delta V = (1\ \text{e})(1\ \text{volt}) = 1.6\times10^{-19}\ \text{J} \equiv 1\ \text{eV}.$$

等位面：相同電位之點所組成之曲面。***a***.場線必垂直於等位面；***b***.在等位面上移動電荷，不需作功；***c***.導體之**表面**(電場沿法線)及**內部**(電場為零)之電位皆相同等；***d***.點電荷之等位面為同心圓。

電荷運動：只受(靜)電力作用時，力學能 E 守恒，

$$E = K + U = \text{const.},\ \Delta E = \Delta K + \Delta U = 0$$

電位：***A***.點電荷 q 之電位或球對稱電荷分佈之外部，$V(r) = k_e\frac{q}{r}$；***B***.點電荷系統 q_i 之電位，$V = \Sigma_i V_i = \Sigma_i k_e\frac{q_i}{r_i}$；***C***.連續電荷分佈—(***a***).由 $dV = k_e\frac{dq}{r}$，得 $V = k_e\int\frac{dq}{r}$，*Note* 有限範圍之電荷分佈才適用，(***b***). 若 $\boldsymbol{E}$ 已知，則 $V_B - V_A = -\int\boldsymbol{E}\cdot d\boldsymbol{s}$。

由電位導得電場：***a***. $dV = -\boldsymbol{E}\cdot d\boldsymbol{s} = -Eds\cos\theta = -E_s ds$，式中 $E_s = E\cos\theta \Rightarrow E_s = -dV/ds$；***b***.用座標表示 $dV = -\boldsymbol{E}\cdot d\boldsymbol{s} = -(E_x dx+E_y dy+E_z dz) \Rightarrow E_x = -(dV/dx)_{y,z=\text{const}} \to -\partial V/\partial x$ 等。*Note* $[E]$ = V/m.

例> 電荷 q，$V(r) = k_e\frac{q}{r}$：***a***. $E_r = -\frac{dV}{dr} = k_e\frac{q}{r^2}$；

b. $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial V}{\partial r}\frac{\partial r}{\partial x} = k_e q\frac{1}{r^2}\frac{x}{r} = k_e q\frac{x}{r^3}$。

電位能：***a***.兩點電荷之電位能 $U_{12} = k_e q_1 q_2/r_{12}$；***b***.點電荷系統 q_i 之電位能 $U = \Sigma_{i\neq j}U_{ij}$，$U_{ij} = k_e q_i q_j/r_{ij}$；***c***.導體(金屬)—球半徑 R 及帶電量 Q：

$$U_e = \frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R}.$$

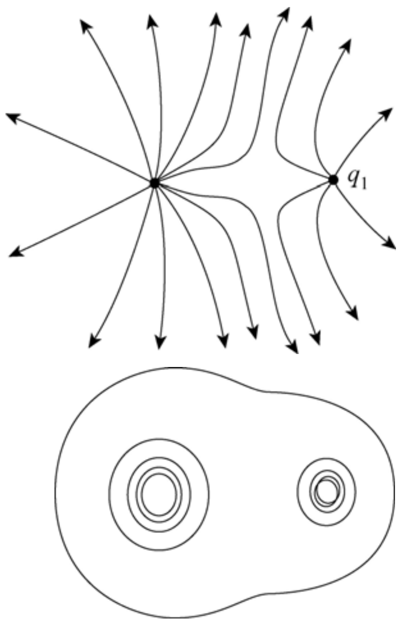
導體球(半徑 R)**表面** $V_R = E_R R = R\sigma/\varepsilon_0$; 金屬表面尖銳點或邊($R$ 小)，電荷密度及電場大。

導體放置於外施電場中，其內自由電子會重新排列而生感應電荷，致使淨電場必垂直導體表面

在這些情況下危險為何？

S3. $\ell n[(x^2+a^2)^{1/2}+x] + \ell n[(x^2+a^2)^{1/2}-x] = \ell n(|a|)$.

99*(a) $q_2 = +2q$ and $q_1 = +q$



99*(b) $q_2 = -3q$ and $q_1 = +q$