Chapter 31 Electromagnetic Oscillations and Alternating Current

2. According to $U = (½)LI^2 = (½)Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00\times10^{-6}}{\sqrt{(1.10\times10^{-3})(4.00\times10^{-6})}} = 4.52\times10^{-2}\ \text{(A)}.$$

6. (**a**) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0}{(2.0\times10^{-3})(0.50)}} = 89\ \text{(rad/s)}.$$

(**b**) The period is $1/f$ and $f = \omega/2\pi$. Therefore,

$$T = 2\pi/\omega = 2\pi/(89\ \text{rad/s}) = 7.0\times10^{-2}\ \text{s}.$$

(**c**) From $\omega = 1/(LC)^{1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89)^2(5.0)} = 2.5\times10^{-5}\ \text{(F)}.$$

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi(LC)^{1/2}$. Consequently,

$$t_{1/4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050)(4.0\times10^{-6})}}{4} = 7.0\times10^{-4}\ \text{(s)}.$$

10. We find the inductance from $f = \omega/2\pi$ and $\omega = 1/(LC)^{1/2}$, $L = 1/(4\pi^2 f^2 C) = 1/[4\pi^2(10\times10^3)(6.7\times10^{-6})] = 3.8\times10^{-5}$ (H).

28. (**a**) We use $I = \varepsilon_m/X_C = \varepsilon_m\omega_d C = 2\pi f_d C\varepsilon_m$:

$$I = 2\pi(1.00\times10^3)(1.50\times10^{-6})(30.0) = 0.283\ \text{(A)}.$$

(**b**) $I = 2\pi(8.00\times10^3)(1.50\times10^{-6})(30.0) = 2.26$ (A).

29. (**a**) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I = \frac{\varepsilon_m}{2\pi f_d L} = \frac{30.0}{2\pi(1.00\times10^3)(50.0\times10^{-3})} = 95.5\ \text{(mA)}.$$

(**b**) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955\ \text{A})/8 = 0.0119\ \text{A} = 11.9\ \text{mA}.$$

39. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0)(70.0\times10^{-6})} = 37.9\ (\Omega).$$

The inductive reactance 86.7 Ω is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(200)^2 + (37.9 - 86.7)^2} = 206\ (\Omega).$$

(**b**) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7 - 37.9}{200}\right) = 13.7°.$$

(**c**) The current amplitude is

$$I = \varepsilon_m / Z = 36.0\ \text{V} / 206\ \Omega = 0.175\ \text{A}.$$

(**d**) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.175\ \text{A})(200\ \Omega) = 35.0\ \text{V},$$
$$V_L = IX_L = (0.175\ \text{A})(86.7\ \Omega) = 15.2\ \text{V},$$
$$V_C = IX_C = (0.175\ \text{A})(37.9\ \Omega) = 6.62\ \text{V}.$$

Note that $X_L > X_C$, so that ε_m leads I. The phasor diagram is drawn to scale below. (Figure)

41. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi,$$

which we solve for R:

$$R = \left(\omega_d L - \frac{1}{\omega_d C}\right)\tan^{-1}\phi = 89\ \Omega.$$

43. (**a**) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf. (**b**) The amplitude of the voltage across the inductor in an RLC series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\ \text{H}}{\sqrt{(1.0\ \text{H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega.$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I_{\text{resonance}} = \varepsilon_m/R = 10\ \text{V}/10\Omega = 1.0\ \text{A}.$$

The voltage amplitude across the inductor is therefore $V_L = IX_L = (1.0\ \text{A})(1000\ \Omega) = 1.0\times10^3$ V, which is much larger than the amplitude of the generator emf.

59. $R = 15.0\ \Omega$, $C = 4.70\ \mu$F, $L = 25.0$ mH, $f = 550$ Hz, and $\varepsilon_m = 75.0$ V. (**a**) The rms current is

$$I_{rms} = 2.586\text{A} \approx 2.59\ \text{A}$$
$$= \frac{\varepsilon_{rms}}{Z} = \frac{\varepsilon_{rms}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}}.$$

(**b**) The rms voltage across R is

$$V_{ab} = I_{rms} R = (2.586)(15.0) = 38.8\ \text{V}.$$

(**c**) The rms voltage across C is

$$V_{bc} = I_{rms} X_C = \frac{I_{rms}}{2\pi fC} = 159.2\ \text{V} \approx 159\ \text{V}.$$

(**d**) The rms voltage across L is

$$V_{cd} = I_{rms} X_L = 2\pi I_{rms} f L = 223.4\ \text{V} \approx 223\ \text{V}.$$

(**e**) The rms voltage across C and L together is

$$V_{bd} = |V_{bc} - V_{cd}| = |159.2\ \text{V} - 223.4\ \text{V}| = 64.2\ \text{V}.$$

(**f**) The rms voltage across R, C and L together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\ \text{V})^2 + (64.2\ \text{V})^2} = 75.0\ \text{V}$$

(**g**) For R, $P_R = \frac{V_{ab}^2}{R} = \frac{(38.8\ \text{V})^2}{15.0\ \Omega} = 100\ \text{W}.$

(**h**) No energy dissipation in C.

(**i**) No energy dissipation in L.

重點整理－第 31 章 電磁振盪與交流電

(同學如有需要,請自行複習第 15 章—力學振盪)

***LC* 能量轉移** 於 LC 振盪電路中，能量於電容器之電場與電感器之磁場間轉移(此類似於木塊 m-彈簧 k 振盪)，兩種形式能量瞬間值

$U_E = \frac{q^2}{2C}$ (vs $U_{sp} = \frac{1}{2}kx^2$), $U_B = \frac{1}{2}Li^2$ (vs $K = \frac{1}{2}mv^2$),

式中 q 為電容器中瞬時電荷，而 i 為通過電感器之瞬時電流．總能量 U $(=U_E + U_B)$仍然固定。

(m-k vs LC: $k \to C^{-1}$, $m \to L$, $x \to q$, $dx/dt \to dq/dt$)

***LC* 電荷與電流振盪** 能量守恆原裡導得 LC 振盪微分方程式 $L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$ (LC 振盪).

其解為 $q = Q\cos(\omega t+\phi)$ (電荷), $i = -\omega Q\sin(\omega t+\phi)$ (電流)，式中 Q 為電荷振幅，而振盪角頻率為 $\omega = 1/\sqrt{LC}$；相位常數 ϕ 由系統初始條件決定。

交流電；強迫振盪 RLC 串聯電路可藉外交流電動勢，$\varepsilon = \varepsilon_m\sin(\omega_d t)$，使其以驅使角頻率 ω_d 作強迫振盪，電動勢驅使的電流，$i = I\sin(\omega_d t-\phi)$，式中 I 及 ϕ 為電流之振幅及相位常數。

單一電路元件 跨過元件之電位差與電流之振幅及相位關係: (***a***)對電阻器，$V_R = IR$，兩者同相；(***b***)對電容器，$V_C = IX_C$，而 $X_C = 1/\omega_d C$ 為**容抗**，電流超前電位差 90°或 $\pi/2$ 相位($\phi = -90°$)；(***c***)對電感器，$V_L = IX_L$，而 $X_L = \omega_d L$ 為**感抗**，電流落後電位差 90°相位($\phi = 90°$)。

Note 電容器與電感器之交流頻率響應截然不同，低頻時，電容器如同斷路，而電感器如同短路；高頻時，電容器如同短路，而電感器如同斷路。

***RLC* 串聯電路** 對具外電動勢，$\varepsilon = \varepsilon_m\sin(\omega_d t)$，之 RLC 串聯電路,(穩定態)電流為 $i = I\sin(\omega_d t-\phi)$，電流振幅 $I = \frac{\varepsilon_m}{Z}$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ 為阻抗，$X_L = \omega_d L$, $X_C = 1/\omega_d C$,而相位常數 $\tan\phi = \frac{X_L - X_C}{R}$.

Note 利用容抗感抗或阻抗，使各元件之電壓與電流之關係如同遵守歐姆定律般。

共振 RLC 串聯電路被正弦變化的外電動勢驅使時，當驅使頻率 ω_d 等於自然角頻率 ω 時，其電流振幅最大，即在**共振**；共振時，電抗等於感抗，$X_C = X_L$，而電流與電動勢同相，$\phi = 0$, $I = \varepsilon_m/R$。

電功率 在 RLC 串聯電路，發電機之平均功率 P_{av} 等於電阻器上熱能產生速率

$$P_{av} = I_{rms}^2 R = \varepsilon_{rms} I_{rms}\cos\phi,$$

上式中下標 rms 表示平方-平均-平方根(方均根)，方均根與最大量(振幅)之關係為 $I_{rms} = I/\sqrt{2}$, $V_{rms} = V/\sqrt{2}$, $\varepsilon_{rms} = \varepsilon_m/\sqrt{2}$, $\cos\phi$ 項稱為電路功率因子.

變壓器 理想變壓器就是匝數 N_p 之原線圈及匝數 N_s 之副線圈纏繞的軟鐵芯．若原線圈與交流發電機連通，原電壓與副電壓的關係為

$V_s / N_s = V_p / N_p$ or $V_s = V_p (N_s/N_p)$ (電壓變換).

流經線圈之電流的關係為

$I_s N_s = I_p N_p$ or $I_s = I_p (N_p/N_s)$ (電流變換).

再者從發電機觀之，副線圈之等效電阻為 $R_{eq} = (N_p/N_s)^2 R$，式中 R 為副電路之電阻負載；比值 N_p/N_s 稱為變壓器匝數比。

太陽爆發如何癱瘓電力網系統？

electromagnetic oscillation 電磁振盪; resonance 共振; alternating current (ac)交流電; direct current (dc)直流電; reactance 電抗; capacitive reactance 容抗,電容性電抗; inductive reactance 感抗,電感性電抗; impedance;阻抗; power factor 功率因子; root-mean-square (rms)方均根; transformer 變壓器; primary/ secondary coil 原/副線圈; turn ratio 匝數比; iron core 鐵芯; (electric) generator 發電機; power grid 電力網; malfunction 發生故障; overtax 超載,負擔過度;

S1.台電提供日常生活用電，電壓 V_{rms} = 110 V or 220 V, 而頻率 f = 60 Hz.

S2.某量 $a(t)$ 與時間關係呈正弦變化 $A\cos(\omega t+\phi)$，則其方均根與振幅關係可藉 $\frac{1}{T}\int_0^T dt\cos^2(\omega t+\phi) = \frac{1}{2}$ 以得.

•**備忘錄**•