Chapter 3 **Vectors Quantities**

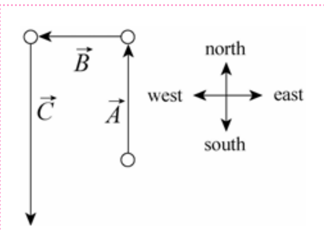
05. The vector sum of the displacements $\boldsymbol{d}_{storm}$ and $\boldsymbol{d}_{new}$ must give the same result as its originally intended displacement $\boldsymbol{d}_0 = 120\,\mathbf{j}$, where east is $\mathbf{i}$, north is $\mathbf{j}$, and the assumed length unit is km. Thus, we write $\boldsymbol{d}_{storm} = 100\,\mathbf{i}$, $\boldsymbol{d}_{new} = A\,\mathbf{i} + B\,\mathbf{j}$. (**a**) The equation $\boldsymbol{d}_{storm} + \boldsymbol{d}_{new} = \boldsymbol{d}_0$ readily yields $A = -100$ km and $B = 120$ km. The magnitude of $\boldsymbol{d}_{new}$ is therefore $(A^2 + B^2)^{1/2} = 156$ km. (**b**) And its direction is $\tan^{-1}(B/A) = -50.2°$ or $180° + (-50.2°) = 129.8°$. We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction ($+x$ axis) and *north* as the $\hat{\mathrm{j}}$ direction ($+y$ axis). All distances are understood to be in kilometers. (**a**) The vector diagram representing the motion is shown below. (**b**) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = -2.4\,\hat{\mathrm{i}} - 2.1\,\hat{\mathrm{j}}.$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4)^2 + (-2.1)^2} \approx 3.2 \text{ (km)}.$$



$$\vec{A} = 3.1\,\hat{\mathrm{j}}$$
$$\vec{B} = -2.4\,\hat{\mathrm{i}}$$
$$\vec{C} = -5.2\,\hat{\mathrm{j}}$$

(**c**) There are two possibilities for the angle:

$$\tan^{-1}\left(\frac{-2.1}{-2.4}\right) = 41°, \text{ or } 221°.$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the $+x$ axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

17. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}$, $\vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the $+x$ axis is 30° + 105° = 135° and apply Eqs.3-5 and 3-6 where appropriate. (**a**) The x component of $\vec{r}$ is $r_x = 10\cos 30° + 10\cos 135° = 1.59$ (m). (**b**) The y component of $\vec{r}$ is $r_y = 10\sin 30° + 10\sin 135° = 12.1$ (m). (**c**) The magnitude of $\vec{r}$ is $(1.59^2 + 12.1^2)^{1/2} = 12.2$ (m). (**d**) The angle between $\vec{r}$ and the $+x$ direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

32. (**a**) With $a = 17.0$ m and $\theta = 56.0°$ we find $a_x = a\cos\theta = 9.51$ m. (**b**) And $a_y = a\sin\theta = 14.1$ m. (**c**) The angle relative to the new coordinate system is $\theta' = (56.0° - 18.0°) = 38.0°$. Thus, $a_x' = a\cos\theta' = 13.4$ m. (**d**) And $a_y' = a\sin\theta' = 10.5$ m.

37. Examining the figure, we see that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$, where $\boldsymbol{a} \perp \boldsymbol{b}$. (**a**) $|\boldsymbol{a}\times\boldsymbol{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º. (**b**) Using the right-hand rule, the vector $\boldsymbol{a}\times\boldsymbol{b}$ points in the $\mathbf{i}\times\mathbf{j} = \mathbf{k}$, or the $+z$ direction. (**c**) $|\boldsymbol{a}\times\boldsymbol{c}| = |\boldsymbol{a}\times(-\boldsymbol{a}-\boldsymbol{b})| = |\boldsymbol{a}\times\boldsymbol{b}| = 12$. (**d**) The vector $-\boldsymbol{a}\times\boldsymbol{b}$ points in the $-\mathbf{i}\times\mathbf{j} = -\mathbf{k}$, or the $-z$ direction. (**e**) $|\boldsymbol{b}\times\boldsymbol{c}| = |\boldsymbol{b}\times(-\boldsymbol{a}-\boldsymbol{b})| = |\boldsymbol{b}\times\boldsymbol{a}| = |\boldsymbol{a}\times\boldsymbol{b}| = 12$. (**f**) The vector points in the $+z$ direction, as in part (a).

39. Since $ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are $a = |\vec{a}| = (3.00^2 + 3.00^2 + 3.00^2)^{1/2} = 5.20$,

$$b = |\vec{b}| = (2.00^2 + 1.00^2 + 3.00^2)^{1/2} = 3.74.$$

The angle between them is found from $\cos\phi =$

$$\frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22°$.

50.• From the figure, it is clear that $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0$, where $\boldsymbol{a} \perp \boldsymbol{b}$. (**a**) $\boldsymbol{a}\cdot\boldsymbol{b} = 0$ since the angle between them is 90º. (**b**) $\boldsymbol{a}\cdot\boldsymbol{c} = \boldsymbol{a}\cdot(-\boldsymbol{a} - \boldsymbol{b}) = -\boldsymbol{a}\cdot\boldsymbol{a} = -a^2 = -16$. (**c**) Similarly, $\boldsymbol{b}\cdot\boldsymbol{c} = -b^2 = -9.0$.

55.• The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\mathrm{i}} + \sin 130°\,\hat{\mathrm{j}}) = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}}$$

and $\vec{B} = B_x\,\hat{\mathrm{i}} + B_y\,\hat{\mathrm{j}} = -7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}$.

(**a**) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}})$$
$$= 5[(-5.14)(-7.72) + (6.13)(-9.20)] = -83.4.$$

(**b**) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\,\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\times$$
$$(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times 10^3\,\hat{\mathrm{k}}.$$

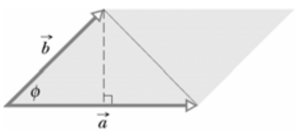
(**c**) Note that the azimuthal angle is undefined for a vector along the z axis. Thus, our result is "1.14 ×

10^3, θ not defined, and $\phi = 0°$." (**d**) Since $\vec{A}$ is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°. (**e**) Clearly, $\vec{A} + 3.00\,\hat{k} = -5.14\hat{i}+6.13\hat{j}+3.00\,\hat{k}$. (**f**) The Pythagorean theorem yields magnitude $A = (5.14^2+6.13^2+3.00^2)^{1/2} = 8.54$. The azimuthal angle is $\theta = 130°$, just as it was in the problem statement [$\vec{A}$ is the projection onto to the xy plane of the new vector created in part (e)]. The angle measured from the $+z$ axis is $\phi = \cos^{-1}(3.00/8.54) = 69.4°$.

(如發現錯誤煩請告知, jyang@mail.ntou.edu.tw, Thanks.)

scalar,純量; vector (sum),向量(和); resultant,合成量; resolving the vector,分解向量; unit vector,單位向量; component,分量; vector/scalar component,向量/純量分量; component notation,分量記法; magnitude-angle notation,大小-角度記法; coordinated system,座標系統; scalar/dot/inner product,純量/點/內乘積; vector/cross/outer product,向量/叉/外乘積; right-hand rule,右手定則; parallelelogram,平行四邊形; (right) triangle,(直角)三角形; base,底邊,基數,壘; altitude,高; hypotenuse,斜邊; diagonal,對角線(的); displacement,位移; east of north,往北偏東; commutative/associate/distributive law,交換/結合/分配律; haphazard,無計劃的,隨意的,雜亂的; landmark,地標;work,功; torque,力矩;

挑戰題•Show that the area of the triangle contained between $\vec{a}$ and $\vec{b}$ and the solid line in right figure is (½)$|\vec{a}\times\vec{b}|$. ***Sol.*** The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b\sin\phi$ and the area is



$$A = \frac{1}{2}ab\sin\phi = \frac{1}{2}|\vec{a}\times\vec{b}|.$$

為由 $\vec{a}$ 及 $\vec{b}$ 相鄰邊組成之平行四邊形的面積之半

***Ex.*1-1**, ***Prob.*3-35** & ***Ex.*1-2**, ***Prob.*3-48**.

重點整理－第 3 章　向　量

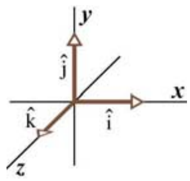
***How does the ant know the way home with no guiding clues on the desert plain?* 螞蟻在沙漠曠野中毫無指引線索下如何知道返巢之路？**

向量: 須要<u>量值</u>及<u>方向</u>才能描述完整的量; 可用指向性之線段以表示, 線段之兩端點: 起點(箭尾)及終點(箭頭), 線段之<u>長度</u>及<u>指向</u>分別表示向量之<u>大小</u>及<u>方向</u>, 如位移,速度,加速度及力等。

向量相加: 性質相同的物理量才能相加。

單位向量: 大小為 1 之向量,

可用以表示方向; 單位向量沒有"單位";



$\hat{i} \parallel x$ axis, $\hat{j} \parallel y$ axis, $\hat{k} \parallel z$ axis,

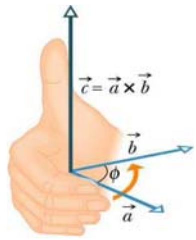
$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$,

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$;

$A = |\vec{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$,

內積 $\vec{A}\cdot\vec{B} \equiv AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$;

向量積 $\vec{A}\times\vec{B} = C\hat{C}$, $C \equiv AB\sin\theta > 0$, where $0 \le \theta \le 180°$, and $\vec{C}\perp\vec{A}$ & $\vec{C}\perp\vec{B}$. $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$, $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$;



$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$

as $\theta = 90° \Rightarrow \vec{A}\cdot\vec{B} = 0$;

as $\theta = 0°$ or $180° \Rightarrow \vec{A}\times\vec{B} = 0$.

坐標軸轉動後, 向量的分量會改變, 但其大小仍不變; 物理量會隨坐標系統而改變, 但物理定律仍保持不變

•"**新一代 GPS**", 艾胥利, 科學人 2003 年 10 月。 • "**GPS：讓路痴不再迷路**", 哈奇森, 科學人 2004 年 6 月。 • "**GPS：讓飛航更安全、更準時**", 翁千婷, 科學人 2004 年 6 月。 •**備忘錄**•