Chapter 4 Two- and Three-Dimensional Motion

04. We choose a coordinate system with origin at the clock center and +*x rightward* (towards the "3:00" position) and +*y upward* (towards "12:00"). (**a**) In unit-vector notation, we have (in cm) $\boldsymbol{r}_1 = 10\,\mathbf{i}$ and $\boldsymbol{r}_2 = -10\,\mathbf{j}$. Thus, Eq.4-2 gives $\Delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = (-10\,\text{cm})\,\mathbf{i} - (10\,\text{cm})\,\mathbf{j}$. Thus, the magnitude is given by $|\Delta\boldsymbol{r}| = [(-10)^2+(-10)^2]^{1/2} = 14\,(\text{cm})$. (**b**) The angle is $\theta = \tan^{-1}(-10/-10) = 45°$ or $-135°$. We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write $\Delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = -10\,\mathbf{i} - 10\,\mathbf{j} \to (14\,\text{cm}\angle{-135°})$. (**c**) In this case, $\boldsymbol{r}_1 = (-10\,\text{cm})\mathbf{j}$ and $\boldsymbol{r}_2 = (10\,\text{cm})\mathbf{j}$, and $\Delta\boldsymbol{r} = (20\,\text{cm})\mathbf{j}$. Thus, $|\Delta\boldsymbol{r}| = 20\,\text{cm}$. (**d**) The angle is given by $\theta = \tan^{-1}(20/0) = 90°$. (**e**) In a full-hour sweep, the hand returns to its starting position, and the displacement is *zero*. (**f**) The corresponding angle for a full-hour sweep is also *zero*.

05. Using Eq.4-3 and Eq.4-8, we have

$$\vec{v}_{av} = \frac{1}{10}[(-2.0\,\hat{\text{i}} + 8.0\,\hat{\text{j}} - 2.0\,\hat{\text{k}}) - (5.0\,\hat{\text{i}} - 6.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}})] = (-0.7\,\hat{\text{i}} + 1.40\,\hat{\text{j}} - 0.40\,\hat{\text{k}})\,(\text{m/s}).$$

11. We apply Eq.4-10 and Eq.4-16. (**a**) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}\,.$$

(**b**) Taking another derivative with respect to time leads to, in SI units (m/s²), $\vec{a} = \frac{d}{dt}(8t\,\hat{\text{j}} + \hat{\text{k}}) = 8\,\hat{\text{j}}$.

22. (跳遠) We use Eq.4-26 and $\theta_0 = 45°$, $R_{max} = v_0^2/g = 9.5^2/9.80 = 9.209 \approx 9.21$ (m), to compare with Powell's long jump; the difference from R_{max} is only $R = 9.21 - 8.95 = 0.259$ (m).

24. (**a**) With the origin at the *initial* point (edge of table), the *y* coordinate of the ball is given by $y = -(½)gt^2$. If *t* is the time of flight and $y = -1.20\,\text{m}$ indicates the level at which the ball hits the floor, then $t = \sqrt{2(-1.20)/(-9.80)} = 0.495$ (s).

(**b**) The initial (horizontal) velocity of the ball is $\boldsymbol{v} = v_0\mathbf{i}$. Since $x = 1.52\,\text{m}$ is the horizontal position of its impact point with the floor, we have $x = v_0\,t$. Thus,

$$v_0 = x/t = 1.52/0.495 = 3.07\ (\text{m/s}).$$

26. (**a**) Using the same coordinate system assumed in Eq.4-22, we solve for $y = h$:

$$h = y_0 + (v_0\sin\theta_0)t - \tfrac{1}{2}g\,t^2,$$

which yields $h = 51.8\,\text{m}$ for $y_0 = 0$, $v_0 = 42.0\,\text{m/s}$, $\theta_0 = 60.0°$ and $t = 5.50\,\text{s}$. (**b**) The horizontal motion is steady, so $v_x = v_{0x} = v_0\cos\theta_0 = 21.0\,\text{m/s}$, but the vertical component of velocity varies according to Eq. 4-23, $v_y = v_0\sin\theta_0 - g\,t = -17.5\,\text{m/s}$. Thus, the speed at impact is $v = (v_x^2 + v_y^2)^{1/2} = 27.4$ m/s.

(**c**) We use Eq.4-24 with $v_y = 0$ and $y = H$:

$$H = (v_0\sin\theta_0)^2/(2g) = 67.5\ \text{m}.$$

35. At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0\cos\theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}(1/5) = 78.5°.$$

39. (射擊之瞄準點) The coordinate origin is taken at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and θ_0 is the firing angle. If the target is a distance *d* away, then its coordinates are $x = R$, $y = 0$. The projectile motion equations lead to $R = v_0 t\cos\theta_0$ and $0 = v_0\,t\sin\theta_0 - (½)gt^2$. Eliminating *t* leads to $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$. Using $\sin\theta_0\cos\theta_0 = (½)\sin(2\theta_0)$, we obtain

$$v_0^2\sin(2\theta_0) = g\,R \Rightarrow \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80)(45.7)}{(460)(460)},$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance *H* above the target, then $\tan\theta_0 = H/R$ so that

$$H = R\tan\theta_0 = (45.7\,\text{m})\tan(0.0606°) = 0.0484\,\text{m} \approx 4.84\,\text{cm}.$$

Solution 2. Owing to the equal height of firing point and target, the falling height *h* from the pointed spot is $h = (½)gt^2$ with *t* being the flight time. We have $t = 45.7/460 = 9.934\times10^{-2}$ (s), $h = \frac{1}{2}g\,t^2 = 0.0484$ (m).

47. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq.4-26. (**a**) We want to know how high the ball is from the ground when it is at $x = 97.5\,\text{m}$, which requires knowing the initial velocity. Using the range information and $\theta_0 = 45°$, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{9.80\times97.5}{1}} = 32.4\ (\text{m/s}).$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{97.5}{32.4\times\cos45°} = 4.26\ (\text{s}).$$

At this moment, the ball is at a height (above the ground) of $y = y_0 + (v_0\sin\theta_0)\,t - (½)g\,t^2 = 9.88$ m, which implies it does indeed clear the 7.32 m high fence. (**b**) At $t = 4.26\,\text{s}$, the center of the ball is $9.88 - 7.32 = 2.56$ (m) above the fence.

58. (UCM) The magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(10\ \text{m/s})^2}{25\ \text{m}} = 4.0\ \text{m/s}^2.$$

60. (衛星) We apply Eq.4-35 to solve for speed *v* and Eq.4- 34 to find acceleration *a*. (**a**) Since the

radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is $6.37\times10^6 + 640\times10^3 = 7.01\times10^6$ (m). Therefore, the speed of the satellite is $v = 2\pi r / T = 2\pi(7.01\times10^6\text{ m})/(98.0\times60\text{ s}) = 7.49\times10^3$ m/s.
(**b**) The magnitude of the acceleration is

$$a = v^2/r = (7.49\times10^3)^2/(7.01\times10^6) = 8.00\ (\text{m/s}^2).$$

61. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (towards the center of the circle) form the basis of this problem. (**a**) If a passenger at this location experiences $\boldsymbol{a} = 1.83\text{ m/s}^2$ east, then the center of the circle is *east* of this location. And the distance is $r = v^2/a = (3.66^2)/(1.83) = 7.32$ (m). (**b**) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west. (**c**) If the direction of $\boldsymbol{a}$ experienced by the passenger is now *south* — indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the *north*.

76. Take velocities to be *constant*; thus, the velocity of the plane relative to the ground is $\boldsymbol{v}_{PG} = 55\text{ km}\,\mathbf{j}/(0.25\text{ h}) = 220\text{ km/h}\,\mathbf{j}$. In addition, $\boldsymbol{v}_{AG} = (42\text{ km/h})(\cos20°\mathbf{i} - \sin20°\mathbf{j}) = (39.4\mathbf{i} - 14.4\mathbf{j})$(km/h). By $\boldsymbol{v}_{PG} = \boldsymbol{v}_{PA} + \boldsymbol{v}_{AG}$, we have $\boldsymbol{v}_{PA} = \boldsymbol{v}_{PG} - \boldsymbol{v}_{AG} = (-39\mathbf{i} + 234\mathbf{j})$ (km/h), which implies $|\boldsymbol{v}_{PA}| = 237$ km/h or 240 km/h.

90.• (**a**) We note that 123° is the angle between the *initial* position and *later* position vectors, so that the angle from $+x$ to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are, respectively,

$$\vec{r}_1 = 360\cos40°\,\hat{\mathrm{i}} + 360\sin40°\,\hat{\mathrm{j}} = 276\,\hat{\mathrm{i}} + 231\,\hat{\mathrm{j}},$$

$$\vec{r}_2 = 790\cos163°\,\hat{\mathrm{i}} + 790\sin163°\,\hat{\mathrm{j}} = -755\,\hat{\mathrm{i}} + 231\,\hat{\mathrm{j}},$$

(in meters). Consequently, we plug into Eq.4-3

$$\Delta\vec{r} = (-755-276)\,\hat{\mathrm{i}} + (231-231)\,\hat{\mathrm{j}} = -(1031\text{ m})\,\hat{\mathrm{i}}.$$

Thus, the magnitude of the displacement $\Delta\boldsymbol{r}$ is $|\Delta\boldsymbol{r}| = 1030$ m. (**b**) The direction of $\Delta\boldsymbol{r}$ is $-\mathbf{i}$, or *westward*.

92.• Eq.4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r. (**a**) The minimum turning radius of the train is given by

$$r_{\max} = \frac{v^2}{a_{\max}} = \frac{(216\text{ km/h})^2}{0.50\times9.80\text{m/s}^2} = 7.3\times10^3\text{ m}.$$

(**b**) The speed of the train must be reduced to no more than

$$v = \sqrt{ra_{\max}} = \sqrt{1000\text{ m}\times0.50\times9.80\text{ m/s}^2} = 22\text{ m/s}.$$

105.• (**a**) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37\times10^6\text{ m})/(8.64\times10^4\text{ s}) = 463\text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = v^2/r = (463)^2/(6.37\times10^6) = 0.034\ (\text{m/s}^2).$$

(**b**) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2R}{T^2}.$$

Thus, $T = 2\pi\sqrt{R/a} = 2\pi\sqrt{(6.37\times10^6)/9.80}$

$$= 5.1\times10^3\text{ (s)} = 84\text{ (min)}.$$

106.• When the escalator is stalled the speed of the person is $v_p = L/t$, where L is the length of the escalator and t is the time the person takes to walk up it. This is $v_p = (15\text{ m})/(90\text{ s}) = 0.167$ m/s. The escalator moves at $v_e = (15\text{ m})/(60\text{ s}) = 0.250$ m/s. The speed of the person walking up the moving escalator is $v = v_p + v_e = 0.167\text{ m/s} + 0.250\text{ m/s} = 0.417$ m/s and the time taken to move the length of the escalator is

$$t = L/v = (15\text{ m})/(0.417\text{ m/s}) = 36\text{ s}.$$

If the various times given are independent of the escalator length, then the answer *does not* depend on that length either. In terms of L (in meters) and the speed (in meters per second) of the person walking on the stalled escalator is $L/90$, the speed of the moving escalator is $L/60$, and the speed of the person walking on the moving escalator is $v = L/90 + L/60 = L/36 = 0.0278L$. The time taken is $t = L/v = L/(0.0278L) = 36$ (s) and is *independent* of L.

116.• Using the same coordinate system assumed in Eq.4-25, we rearrange that equation to solve for the initial speed: $v_0 = \dfrac{x}{\cos\theta_0}\sqrt{\dfrac{g}{2(x\tan\theta_0 - y)}}$,

which yields $v_0 = 23$ ft/s for $g = 32$ ft/s², $x = 13$ ft, $y = 3$ ft, and $\theta_0 = 55°$.

118.• Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and $v_y = 0$ at the target, we obtain $v_{0y} = [2(9.80)(5.00)]^{1/2} = 9.90$ (m/s). (**a**) Since $v_0\sin\theta_0 = v_{0y}$, with $v_0 = 12.0$ m/s, we find $\theta_0 = 55.6°$. (**b**) Now, $v_y = v_{0y} - gt$ gives $t = 9.90/9.80 = 1.01$ (s). Thus, $x = (v_0\cos\theta_0)t = 6.85$ m. (**c**) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0\cos\theta_0 = 6.78$ m/s.

120.• With $v_0 = 30.0$ m/s and $R = 20.0$ m, Eq.4-26 gives $\sin2\theta_0 = gR/v_0^2 = 0.218$. Because $\sin(180°-\theta) = \sin\theta$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58°\text{ and }167.4°,$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects). (**a**) The smallest angle is $\theta_0 = 6.29°$. (**b**) The greatest angle is and $\theta_0 = 83.7°$. An alternative approach to this problem in terms of Eq.4-25 (with $y = 0$ and $\sec^2\theta = 1 + \tan^2\theta$) is possible — and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

***Ex.*3-1**, ***Prob.*4-15** & **4-23**; ***Ex.*3-2**, ***Prob.*4-46**; ***Ex.*3-3**, ***Prob.*4-67**& **4-75**.

重點整理－第 4 章 二維與三維運動

二維(平面)運動：即兩互不影響的“**水平**”方向與“**鉛直**”方向運動之向量合成。(底下 $g>0$)

抛體運動：初速度: (v_0, θ_0)或$(v_0\cos\theta_0, v_0\sin\theta_0)$

水平(x)：<u>等速</u> $a_x=0$ 及鉛直(y)：<u>等加速</u> $a_y=-g$,

速度：$v_x=v_0\cos\theta_0$, $v_y=v_0\sin\theta_0-g\,t$, ($\Delta y>0$: *up*)

位置：$x-x_0=v_0\cos\theta_0\,t$, $y-y_0=v_0\sin\theta_0\,t-\frac{1}{2}g\,t^2$,

路徑為抛物線: $y=(\tan\theta_0)\,x-\dfrac{g\,x^2}{2(v_0\cos\theta_0)^2}$, for $x_0=y_0=0$. ◆抛體於最高點時, 鉛直速度為零。

當出發與落地高度相等($y=y_0$)時, **<u>水平射程</u>**, $R=\dfrac{v_0^{\,2}}{g}\sin(2\theta_0)$, 當 $\theta_0=45°$, $R_{max}=\dfrac{v_0^{\,2}}{g}$；而 θ_0 由 0 增至 45°時, R 變大; 但 θ_0 由 45°增至 90°時, R 變小; 當 v_0 及 g 固定下, 有兩 θ_0 (兩者和= 90°)可得相同的 R.

<u>最大爬升高度</u> $H=\dfrac{(v_0\sin\theta_0)^2}{2g}$, v_0 變大, H 變大; θ_0 增加, H 亦變大。**<u>上升時間</u>** = **<u>下降時間</u>**, **<u>飛行時間</u>** $T=\dfrac{2v_0\sin\theta_0}{g}$, θ_0 增加, T 變大; v_0 變大, T 亦變大。

kinematics,運動學; dynamics,動力學; mechanics,力學; parabola,抛物線; hyperbola,雙曲線; ellipse,橢圓; projectile motion,抛體運動; initial velocity,初速; launching,發射/下水; landing,降落/登陸; equation of the path,路徑方程式; trajectory,彈/軌道; horizontal range,水平射程; uniform circular motion,等速率圓周運動; centripetal acceleration,向心加速度; reference frame,參考(座標)系; relative motion,相對運動; Track & Field,田徑; sprinter,短跑選手; high jump,跳高; long jump,跳遠; shot put,推鉛球; free-throw/foul line,罰球線; three-point line,三分球線; overhand push shot,過肩推球投籃; underhand loop shot,腰間弧射投籃; pivot shot,轉身投籃; dunk shot,灌籃; ramp,坡道; merry-go-round,旋轉木馬; Ferris wheel,摩天輪; roller coaster,雲霄飛車; loop-the-loop,翻跟斗; pirate,海盜; dog-fight,空戰; Top gun,捍衛戰士; tunnel vision,視覺極端窄化; g-LOC, g 誘發意識喪失; barrel,槍管; skateboard,滑板; time of flight,飛行時間;

What clue is hidden in ball's motion?

棒球運動隱藏什麼線索？

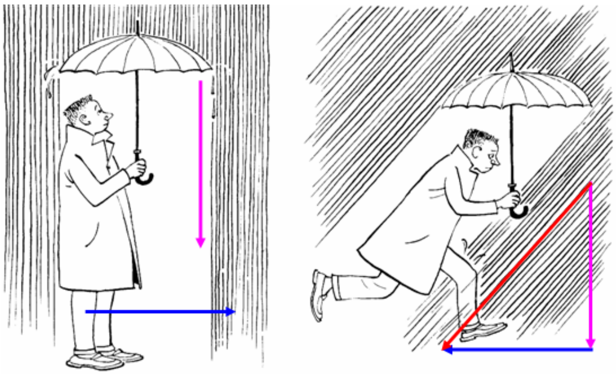
(人站對位置, 看球爬升角度以定速率增加)

等速率圓周運動：質點以等速率 v 於圓周(半徑 r)上運動稱之；**週期** $T=2\pi r/v$ (or $v=2\pi r/T$), (**向心**)加速度方向永指向圓心，**加速度大小**

$$a\,(a_c)=\frac{v^2}{r}=\frac{4\pi^2 r}{T^2}\ \Leftarrow\ vT=2\pi r=\text{圓周長}.$$

相對運動：A, B: 參考點或觀測者, P: 質點,

位置 $\vec r_{PA}=\vec r_{PB}+\vec r_{BA}$，速度 $\vec v_{PA}=\vec v_{PB}+\vec v_{BA}$ 。



(雨鉛直落下, 該如何撐傘? 懂物理就勿須煩惱!)

Q.抛體在鉛直面上運動的路線為一抛物線; 這表示 (a) 沒有加速度 (b) 加速度是固定的 (c)在 x 及 y 方向上有不同大小的加速度 (d) (b)和(c)皆是.

Q.若某抛體之最大飛行高度為水平射程的 3 倍, 試計算此抛體之抛射角度 θ_0, 設出發與落地高度一樣。**Q**.(*Prob.35*)若某抛體於最大飛行高度時的速率為其抛射速率的一半，試計算此抛體之抛射角度 θ_0.

挑戰題〉**1**.設出發高度與觸地高度相同，某抛體於空中運動之最大爬升高度為 H, 而水平射程為 R, 試計算(**a**)此抛體之抛射(或出發)速度及(**b**)於空中飛行時間。**2**.如何跳遠才能跳得遠. **3**.鉛球如何推才能推得遠；**4**.籃球如何投才能百發百中/最省體力. (有興趣,請上教學網站) •有關空氣阻力對抛體之效應，請參閱“**牛頓打棒球**”，李靜宜譯，牛頓。 •**備忘錄**•