Chapter **10** (Benson)

E04 $(R/2)^2(R/2) = [R^2-(R/2)^2]\,x_c$, $x_c = R/6$ (left of center) .

E06 $x_{cm} = [0 - \rho\,(4\pi/3)r^3 d\,]/[\rho\,(4\pi/3)(R^3 - r^3)] = -r^3 d/(R^3 - r^3)$.

E13 (a) $x_c = [(1)(L/2) + (1)(L/4) + (1)(3L/4)]/3 = L/2$. $y_c = [0 + (2)(\sqrt{3}L/4)]/3 = \sqrt{3}L/6 = 0.29\,L$. (b) $x_c = [(1)(L/2) + (\sqrt{2})(L/2)]/(2 + \sqrt{2}) = (1 + \sqrt{2})L/(4 + 2\sqrt{2}) = 0.354\,L$. $y_c = [(1)(L/2)+(\sqrt{2})(L/2)]/(2+\sqrt{2}) = 0.354\,L$.

E15 $m\,(-L)+(2m)L+3m\,(0) = (6m)\,x_c$, $x_c = L/6$. $m\,(-2L+\Delta x)+3m\,(\Delta x) + 2m\,(2L+\Delta x) = (6m)\,x'_c$. As $x_c = x'_c$, $\Delta x = -L/6$.

E18 $m_1 = 2\,\text{kg}$, $\vec{r}_1 = (2\,\hat{i} + 3\,\hat{j})\text{m}$ & $\vec{v}_1 = (-\hat{i} + 5\,\hat{j})\,\text{m/s}$; $m_2 = 5\,\text{kg}$, $\vec{r}_2 = (-5\,\hat{i} + \hat{j})\text{m}$ & $\vec{v}_2 = (3\hat{i} - 4\hat{j})\,\text{m/s}$. (a) $\vec{r}_{cm} = (-21\hat{i} + 11\hat{j})/7 = (-3\hat{i} + 1.57\hat{j})\,(\text{m})$; (b) $\vec{v}_{cm} = (13\hat{i} - 10\hat{j})/7 = (1.86\hat{i} - 1.43\hat{j})$ (m/s); (c) $\vec{p} = (13\hat{i} - 10\hat{j})\,\text{kg·m/s}$; (d) $\vec{R}_2 = \vec{R}_1 + \vec{v}_{cm}\Delta t = (0.72\hat{i} - 1.29\hat{j})\,\text{m}$.

E22 $\vec{P} : (60+1)(2\,\hat{i}) = (1)(5\,\hat{j}) + 60\,\vec{v}$, $\vec{v} = 2.03\,\hat{i} - 0.083\hat{j}$ (m/s); (b) $\Delta r_c = \vec{v}_c\Delta t = (2\,\hat{i})(4) = 8\,\hat{i}$ (m) (east) .

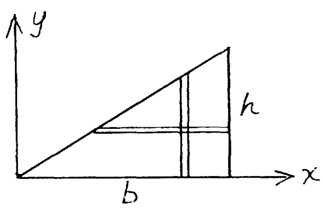
E25 (a) $\vec{V}_c = [(0.8)(3\,\hat{i}) + (1.2)(-5\,\hat{i})]/(0.8 + 1.2) = -1.8\,\hat{i}$; (b) $\vec{V}_1 - \vec{V}_c = 3\,\hat{i} - (-1.8)\,\hat{i} = 4.8\,\hat{i}$, $\vec{V}_2 - \vec{V}_c = -5\,\hat{i} - (-1.8)\,\hat{i} = -3.2\,\hat{i}$; (c) $K_t = (0.8)(3)^2/2 + (1.2)(5)^2/2 = 3.6 + 15 = 18.6$(J); (d) $K_{cm} = (0.8 + 1.2)(1.8)^2/2 = 3.24$(J); (e) $K_{rel} = (0.8)(4.8)^2/2 + (1.2)(3.2)^2/2 = 9.22 + 6.14 = 15.36$ (J) .

E28 (a) $v_{cm} = v/21$, $K_{rel} = 0.5(1u)(20v/21)^2 + 0.5(20u)(v/21)^2 = 10uv^2/21$. Thus $E = 10uv^2/21 = 8 \times 10^{-19}$ J. $K_i = 0.5(1u)v^2 = 8.4 \times 10^{-19}$ J; (b) $v_{cm} = 20v/21$, $K_{rel} = 0.5(1u)(20v/21)^2 + 0.5(20u)(v/21)^2 = 10uv^2/21$. Thus $E = 10uv^2/21$. $K_i = 0.5(20u)v^2 = 168 \times 10^{-19}$ J . (Teacher: Jyh-Shinn Yang, 89.12.20)

E32 (a) $F_{thrust} = V_{ex}dm/dt = (2600)(1.5\times10^4) = 3.9\times10^7$ (N); (b) $F_{thrust} - mg = ma$, $3.9 \times 10^7 - (2.5 \times 10^6)(9.8) = (2.5 \times 10^6)\,a$, $a = 5.8\,(\text{m/s}^2)$.

E39 (a) $V_c = (m_1\vec{V}_1 + m_2\vec{V}_2)/(m_1 + m_2) = 16.51\,\hat{i}\,(\text{m/s})$; (b) $\vec{V}'_1 = -2.5\,\hat{i}\,(\text{m/s})$, $\vec{V}'_2 = -5.5\,\hat{i}\,(\text{m/s})$ (c) $\vec{P}'_1 = m_1\vec{V}'_1 = -6875\,\hat{i}$ (kg.m/s), $\vec{P}'_2 = 6875\,\hat{i}$ (kg.m/s) .

P01 $y = hx/b$, $dm = \sigma y dx = \sigma h x dx/b$ & $M = \sigma bh/2$. $\int x dm = (\sigma h/b)\int x^2 dx = \sigma h b^2/3$. $\int y dm = \int \sigma y(b - x)dy = \int \sigma(by - by^2)dy/h = \sigma b h^2/6$. Thus $x_c = 2b/3$, $y_c = h/3$.



P02 From Exa. 10.3, $y_{cm} = 2r/\pi$ for a semi-circular ring. $y_{cm} = \int y dm/M = \int(2r/\pi)\sigma\pi r dr/(\sigma\pi R^2/2) = 4R/(3\pi) = 0.424\,R$.

P05 Let the apex of the cone be at the origin. $M = \int dm = \int 2\pi\sigma r\,dz \csc\alpha$, where $r = z\tan\alpha$. $M = 2\pi\sigma\tan\alpha\csc\alpha h^2/2$. $Mz_{cm} = \int z\,dm = 2\pi\sigma\tan\alpha\csc\alpha h^3/3$. Thus $z_{cm} = 2h/3$.

P08 m_1(=50kg), m_2(=25kg), and m_3(=5kg) are the masses of the boy, platform, and ball.

(a) $(2m_1 + m_2 + m_3)\,u = (2m_1 + m_2)\,v + m_3\,(u + v)$, $(130)(2) = (125)\,v + 5(4 + v)$, $v = 24/13 = 1.85$(m/s); (b) $S = vt = v(L/u) = (14/13)(4/4) = 1.85\,$(m); (c) $S_{cm} = v_{cm}t = (2)(1) = 2\,$(m) .

P09 (a) $F_{thrust} = V_{ex} dm/dt = (2000)(100) = 2.0\times 10^5$ (N); (b) $V - V_0 = V_{ex} \ln(M_0/M)$, $M = M_0 - (dm/dt)\,t = 5.0\times 10^4 - (100)(30) = 4.7\times 10^4$ (kg), $V = 1.0\times 10^3 + (2000)\ln(50/47) = 1.12\,$(km/s) .

P12 $mu = (M + m)V_c$, $K_{rel} = m(u - V_c)^2/2 + MV_c^2/2 = mMu^2/[2(M + m)]$. $K_{rel} \geq E$, thus $mu^2/2 \geq (M + m)E/M$. (Teacher: Jyh-Shinn Yang, 89.12.20)