Chapter **15** (Benson)

E07 (**a**) $m=0.5\,\text{kg}$ & $k=50\,\text{N/m} \Rightarrow \omega=10\,\text{rad/s}$. $x=A\sin(10t+\phi)$, $v=10A\cos(10t+\phi)$; $x=-0.2\,\text{m}$ & $v=+0.5\,\text{m/s}$ at $t=0.1\,\text{s}$. Then $A^2=x^2+(v/10)^2=0.0424$, $A=0.206\,(\text{m})$. $\tan(1+\phi)=10x/v=-4.0$, $1+\phi=-1.32$, $\phi=-2.33$; (**b**) $x=0.206\sin(10t-2.33)\,\text{m}$; (c) $\tan(10t-2.33)=-4.0$, $10t-2.33=1.81$, $t=0.414\,\text{s}$.

E09 (**a**) $k=mg/\Delta y=30.6\,\text{N/m}$ & $m=0.5\,\text{Kg} \Rightarrow \omega=\sqrt{k/m}=7.83\,\text{rad/s}$. Since $y=A=0.08\,\text{m}$ at $t=0$, $\phi=\pi/2$. So $y=0.08\sin(7.83t+\pi/2)\,\text{m}$; (**b**) $y=-0.06\,\text{m} \Rightarrow \cos(7.83t)=-0.75$ & $|\sin(7.83t)|=0.661$, then $|v|=\omega A\sin(\omega t)=0.414\,\text{m/s}$, $a=-\omega^2 y=+3.68\,\text{m/s}^2$.

E12 $T=2\pi\sqrt{m/k_e}$. (**a**) $F=k_1x_1=k_2x_2=(x_1+x_2)k_e$; $F/k_e=x_1+x_2=F/k_1+F/k_2$, so $k_e=k_1k_2/(k_1+k_2)$; (**b**) & (**c**): $k_e=k_1+k_2$.

E16 $m=10^{-26}\,\text{Kg}$, $f=10^{12}\,\text{Hz}$ & $A=0.05\,\text{nm}$. (**a**) $v_m=2\pi fA=314\,\text{m/s}$, $E=K_m=mv_m^2/2=4.93\times10^{-22}\text{J}$; (**b**) $314\,\text{m/s}$; (**c**) $a=(2\pi f)^2A=1.97\times10^{15}\text{m/s}^2$; (**d**) $k=(2\pi f)^2 m=0.395\,\text{N/m}$.

E18 A (**a**), E (**b**), and ϕ (**d**) are unchanged; (**b**) $T_2/T_1=\sqrt{m_2/m_1}=\sqrt{3/2}$.

E21 $kx^2/2+mv^2/2=kA^2/2 \Rightarrow v=\pm\sqrt{(k/m)(A^2-x^2)}$.

E23 $T=2\pi\sqrt{I/mgd}$. (**a**) $I=M/3 \Rightarrow T=1.64\,\text{s}$; (**b**) $I=7M/75 \Rightarrow T=1.94\,\text{s}$.

E24 $T=2\pi\sqrt{I/Mgd}=2\pi\sqrt{3R/2g}$.

E38 $m=0.2\,\text{kg}$, $v_m=1.25\,\text{m/s}$ & $a_m=9\,\text{m/s}^2$ (**a**) $v_m=\omega A$ & $\omega^2A=a_m \Rightarrow \omega=7.2$ rad/s & $A=0.174\,\text{m}$; (**b**) With $k=m\omega^2$ & $x=0.12\,\text{m}$, then $kA^2/2=kx^2/2+mv^2/2$ gives $v=0.907\,\text{m/s}$.

E58 $E=0.18\,\text{J}$, $A=114\,\text{cm}$ & $v_m=1.25\,\text{m/s}$ (**a**) $E=mv_m^2/2$ gives $m=0.230\,\text{m/s}$; (**b**) $E=kA^2/2$ gives $k=18.4\,\text{N/m}$; (**c**) $\omega=v_m/A$ gives $f=\omega/2\pi=1.42\,\text{Hz}$; (**d**) With $x=7\,\text{cm}$, $E=kx^2/2+mv^2/2$ gives $v=1.08\,\text{m/s}$.

E63 $l=gT^2/4\pi^2=0.993\,\text{m}$. (Teacher: Jyh-Shinn Yang, 90.03.07)

E65 $T=2\pi\sqrt{I/Mgd}=2\pi\sqrt{2L/3g}$, so $l=2L/3=0.8\,\text{m}$.

E79 l_1, $l_2=81$, $64\,\text{cm} \Rightarrow T_2/T_1=8/9$. $(n+1)T_2=nT_1$ gives $n=8$ & $nT_1=14.5\,\text{s}$.

E87 (**a**) $v=0.5v_m \Rightarrow K=0.25E$, & $U=0.75E$. $kx^2/2=0.75(kA^2/2)$, $x=0.866A=17.3\,\text{cm}$; (**b**) $K=U$, $U=0.5E$, $x^2=0.5A^2$, $x=14.1\,\text{cm}$.

P02 $mg-N=ma_y$, $N=mg-ma_y\geq 0$. As the coin loses contact with the piston, $a=\omega^2A\geq g$ $\Rightarrow f_{min}=\sqrt{g/A}/2\pi=1.58\,\text{Hz}$.

P03 $E=ky^2/2-mgy+mv^2/2+(MR^2/2)(v/R)^2/2=\text{const}$. Set $dE/dt=0$ to find $(m+M/2)d^2y/dt^2+k(y-mg/k)=0$. Using $y_1=y-mg/k$, we have $d^2y_1/dt^2+2ky_1/(M+2m)=0$, or $\omega^2=2k/(M+2m)$.

P04 $m = 1\,\text{kg}$, $M = 5\,\text{kg}$, $k = 20\,\text{N/m}$ & $A = 0.4\,\text{m}$. $\omega^2 = k/(M+m)$, $a = \mu g$ for m, $a_m = \omega^2 A \leq \mu g$, thus $\mu \geq 0.136$.

P05 $\tau = I\alpha$: $-(mg\sin\theta)R = (MR^2)(d^2\theta/dt^2)$ leads to $\omega = \sqrt{g/R}$.

P06 (**a**) $F = ma$: $(lA\rho)d^2x/dt^2 = -2xA\rho g$, $d^2x/dt^2 = -(2g/l)x$, i.e. SHM. (**b**) $T = 2\pi\sqrt{l/2g}$.

P08 Let y be the downward displacement, $F_y = -\rho_f Ayg = (\rho_b Ah)d^2y/dt^2$.
$d^2y/dt^2 = -(\rho_f g/\rho_b h)\,y$, so $\omega^2 = \rho_f g/\rho_b h$.

P12 (**a**) $E = mv^2/2 + kx^2/2 - mgy$; (**b**) $dE/dt = 0 \Rightarrow md^2y/dt^2 + kx - mg = 0$; (**c**) Let $y' = y - y_0 = y - mg/k$, then $md^2y'/dt^2 + ky' = 0$, with $\omega^2 = k/m$.

P13 $F_x = -F_g \sin\theta = -(mgr/R)(x/r) = ma_x$. Thus $\omega^2 = g/R$ and $T = 2\pi/\omega \simeq 84.4\,\text{min}$.

P14 $\tau = I\alpha$: $I\alpha = (-kx)\,L$. $x = L\theta$ & $I = ML^2/3$. Find $d^2\theta/dt^2 + 3k\theta/M = 0$, thus $T = 2\pi\sqrt{M/3k}$.

(Teacher: Jyh-Shinn Yang, 90.03.07)